The Modified Partial Healing Model used as a Prediction Tool for the Complex Stiffness Modulus Evolutions in Four Point Bending Fatigue Tests based on the Evolutions in Uni-Axial Push-Pull Tests.

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ABSTRACT: The Partial Healing (PH) model has proven to be a good tool in describing the evolution of the (weighted) complex stiffness modulus for a beam in a four point bending continuous fatigue test in controlled deflection mode. In principle the PH model is a material model which describes the change in the complex stiffness modulus for a unit volume due to loading. Taking into account the geometrical dimensions of the tested specimen the evolution of the weighted complex stiffness modulus for the specimen can be calculated when the parameters of the PH material model are known. The stress-strain field in uni-axial push-pull (UPP) tests ought to be homogenous over the length and the cross area of the cylinder. So, the evolution of the complex stiffness modulus in the UPP test can be used directly for the determination of the (material) parameters of the PH model. Therefore UPP tests can be used to predict the evolution of the complex stiffness modulus in two point bending (2PB) and four point bending (4PB) tests which can be compared with the measured evolution. This procedure is applied to 4PB test results of a RILEM project using a modified PH model. The modified PH model takes into account the possible existence of the so called low endurance limit. In the RILEM project several devices were used to investigate the fatigue properties of the same mix. The results for the UPP and 4PB tests are presented in this paper together with a new view on the definition and determination of the fatigue life. This might open a way for a comparison of the fatigue lives measured with different devices.

KEY WORDS: Fatigue, endurance limit, healing, stiffness modulus evolution, fatigue life.

1 INTRODUCTION

A major criterion in the design of asphalt pavements is the fatigue characteristics of an asphalt mix. The determinations of the fatigue characteristics are often carried out with dynamic cyclic bending tests with two, three or four point bending (2PB, 3PB, 4PB) devices applying constant sinusoidal deflections at a certain frequency and at a constant temperature.

Force, deflection and the phase lag between force and deflection are measured during the test and are used for the calculation of the beam stiffness modulus. Normally the fatigue life is defined as the number of cycles at which the beam stiffness modulus has decreased to half its initial value. Mainly because the response of the specimen (beam) is measured and not the response of the material, the fatigue life determined with different devices will differ even between two similar devices but with different geometries for the specimens. This lack of agreement can be avoided if a material response model is the starting point of the back calculation. By taking into account the differences in configuration and stress/strain distributions in the specimen the response of the tested specimen can be determined.

Another issue is the phenomenon of healing. Rest periods during a fatigue test can increase the fatigue life (Bonnaure et al. 1982, Francken, 1978, Kim et al. 1989, Kim et al. 1994, Pronk, 1997). In these investigations the effect of healing was studied by applying real rest periods. However, if instead of a real rest period a pseudo rest period is applied with lower strain amplitude it has been shown that the stiffness also increases during these pseudo rest periods (Pronk, 1997). This implies that the phenomenon of reversible damage should be directly incorporated in the interpretation of fatigue tests with respect to the evolution of the stiffness modulus and phase lag.

Therefore a material model should be used that implicitly includes the phenomenon of healing. Such a model is the Partial Healing (PH) model (Pronk, 2001).The equations of the PH model are believed to describe the evolution of the complex stiffness modulus (modulus and phase lag) during the fatigue test satisfactorily.

2 THEORY

The PH model is in principle a material model that describes the evolution of the properties for a unit volume. After integration over the dimensions of the specimen, taking into account the configuration (spatial strain/stress distribution) of the test device, a weighted complex stiffness modulus for the specimen is obtained. Given the complexity of the equations in the PH model this integration has to be carried out numerical.

It is assumed that next to the viscous-elastic dissipated energy ΔW_{dis}, which is completely transformed into heat, also energy is dissipated for the creation of micro cracks, dislocations and other defects. In strain controlled fatigue tests with sinusoidal loading the dissipated viscous-elastic dissipated energy per cycle is given by equation 1 in which ε_0 is the constant strain, σ is the stress, S is the stiffness modulus, φ is the phase lag between σ and ε and F represents the loss modulus.

$$\Delta W_{dis} = \pi\varepsilon\sigma\sin(\phi) = \pi\varepsilon_0^2 S\, sin\left(\varphi\right) = \pi\varepsilon_0^2 F \qquad (1)$$

The assumption is that the damage Q is considered to have the same formulation as ΔW_{dis} (Pronk, 1990) which leads for strain controlled tests to equation 2 in which δ is a small parameter and T is the reciprocal value of the applied frequency f.

$$\frac{d}{dt}Q = \frac{d}{dt}\delta.W_{dis} \approx \delta\frac{\Delta W_{dis}}{T} = \delta\, f\,\pi\,\varepsilon_0^2\, F \qquad (2)$$

The damage Q affects both the loss modulus F and the storage modulus G during loading according to equations 3 and 4. The first term in the integrals with the parameters $\alpha_{1,2}$* and β represents damage which heals in time (reversible damage) and the second term with the parameter $\gamma_{1,2}^*$ represents the irreversible damage which will just accumulate during the fatigue test.

$$F\{t\} = F_o - \int_0^t \frac{dQ\{\tau\}}{d\tau}\left[\alpha_1^* e^{-\beta(t-\tau)} + \gamma_1^*\right]d\tau \qquad (3)$$

$$G\{t\} = G_o - \int_0^t \frac{dQ\{\tau\}}{d\tau}\left[\alpha_2^* e^{-\beta(t-\tau)} + \gamma_2^*\right]d\tau \qquad (4)$$

The source of the reversible damage is not really known. In the author's view it could be a kind of pseudo plasticity like thixotropy or a thin liquid behavior. Nevertheless, equations 3 and 4 describe the measured evolutions very well.

Fitting the PH model on 4PB fatigue tests at different strain levels showed that the parameter β is dependent on the squared value of the applied strain ($\beta=\beta^{*}\varepsilon_0^{2}$).

Due to the transformation of the dissipated viscous-elastic energy per cycle (equation 1) into heat the temperature of the tested specimen will increase and as a consequence the modulus of the complex stiffness modulus will decrease and the phase lag will increase. This phenomenon is not taken into account in the PH model. But if forced convection temperature control is used, the temperature increase is limited in 4PB and UPP tests (Pronk, 1996). It should be marked that for the asphalt mix at issue the coefficient α_1^{*} for the reversible damage of the loss modulus F turned out to be nil. This simplifies the equations considerable.

3 UNI-AXIAL PUSH-PULL TESTS

As mentioned above the PH model is a material model. Therefore a direct application of the PH model is only possible for the UPP tests. In a RILEM project (Di Benedetto et al. 2004) fatigue tests were carried out on the same mix using different fatigue devices. The results for the UPP and 4PB tests are used in this paper. In the UPP test the evolution of the complex stiffness modulus for any unit volume should be equal to the evolution for the whole cylinder. However, a constant uniform strain field throughout the whole UPP cylinder is difficult to maintain. Normally three strain gages or LVDT's, which are equally located around the specimen (120 degrees), are used for the process control. In practice only in 20% of the tests the strains measured by the three sensors can be considered equal to each other (mean differences less than 5 μm/m). Here only two UPP tests can be considered as real constant strain tests (E13D80-14 and E13D180-17; table 1). Nevertheless taking the mean strain value it appears that the evolution of the complex stiffness modulus in the other tests could also be fitted very well with the PH model. The fit is carried out starting with cycle 1000 in order to eliminate partly the relatively large temperature effect in the initial phase of the test. The end of the regression interval was taken equal to the fatigue life N1 (Hopman et al. 1989). In total five UPP tests were available. In table 1 the mean values of the strains in this interval are given together with the range and if the strains decreased or increased on this interval.

Table 1: Strain amplitudes on the interval from 1000 cycles to N1 and the variation in strain amplitudes (random, increase or decrease) on this interval.

Beam	Average [μm/m]	Minimum [μm/m]	Maximum [μm/m]	Change
E13D 80 – 14	81.9	79.3	83.4	random
E13D 180 – 17	173.6	170	176	↓
E13D 100 – 01	115	109	126	↑
E13D 100 – 10	159.4	151	179	↑
E13D 100 – 13	111.2	107	120	↑

The values for the regression coefficients are given in table 2 if a mean strain value is adopted. Even for an increase of 28 μm/m on the regression interval (E13D100-10) the fit is good as indicated in figure 1. Reversible damage in the loss modulus was taken into account in the PH model, but it appeared that at least for this asphalt mix this term is nil ($\delta\alpha_1^{*} = 0$).

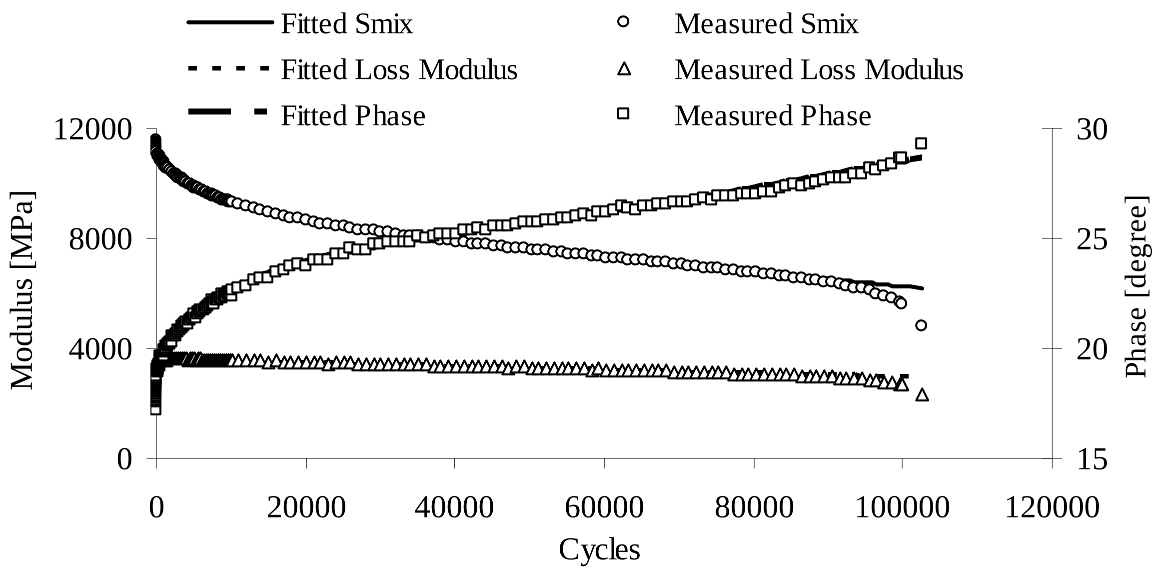

Figure 1: Measured and fitted evolution of the complex stiffness modulus for cylinder E13D100-10 in the UPP test while using a mean strain value of 159.4 μm/m.

Table 2: PH parameter values from regressions adopting mean strain values on the interval.

Beam	$\delta\alpha_1^*=\alpha_1/(\pi f\varepsilon^2)$	$\delta\alpha_2^*=\alpha_2(\pi f\varepsilon^2)$	$\delta\gamma_1^*=\gamma_1/(\pi f\varepsilon^2)$	$\delta\gamma_2^*=\gamma_1/(\pi f\varepsilon^2)$	$\beta^*=\beta/\varepsilon^2$	S_{mix}	φ
	[-]	[-]	[-]	[-]	[s^{-1}]	[GPa]	[^{o}]
E13D 80-14	0	242.8	3.4	15.0	22200	10.5	18.5
E13D100-01	0	731.6	7.5	52.5	51200	12.3	17.9
E13D100-10	0	856.0	25.0	121.8	50600	10.9	19.4
E13D100-13	0	740.7	7.5	45.1	51500	11.4	17.9
E13D180-17	0	1111.1	47.9	136,3	54700	11.2	20.9

The individual fits are good as shown in figure 1, but the fitted values for the PH parameters in table 2, especially the parameters $\delta\gamma_1^*$ and $\delta\gamma_2^*$, are quite different from each other, which is not in accordance with the starting point in the 'theory'. In the theory it is adopted that the parameters $\delta\alpha_1^*$, $\delta\alpha_2^*$, $\delta\gamma_1^*$, $\delta\gamma_2^*$ and β^* are constants. However, it's not unlikely that both the reversible and irreversible damage will increase with a higher strain level. In view of the results given in table 2 and the good comparison between regression and measured evolution, the time decay constant β^* can be taken equal to 52,000 [s^{-1}] without introducing large deviations.

Table 3: PH parameter values if the parameter α_1 is taken equal to 0 and β equal to 50,000 s^{-1}.

Beam	Average ε	$\delta\alpha_2^*=\alpha_2/(\pi f\varepsilon^2)$	$\delta\gamma_1^*=\gamma_1/(\pi f\varepsilon^2)$	$\delta\gamma_2^*=\gamma_1/(\pi f\varepsilon^2)$
	[μm/m]	[-]	[-]	[-]
E13D80-14	81.9	579.3	3.4	16.4
E13D100-13	111.2	748.5	7.5	45.2
E13D100-01	115	742.9	7.5	52.6
E13D100-10	159.4	875.4	25.0	123.0
E13D180-17	173.6	1060.1	47.9	133.8

A regression with these adoptions leads to the PH parameter values given in table 3. The renewed regression did not lead to larger deviations between the fitted and the measured evolutions as indicated by figure 2 for beam E13D80-14. The obtained coefficients seem to depend on the applied strain level (table 3).

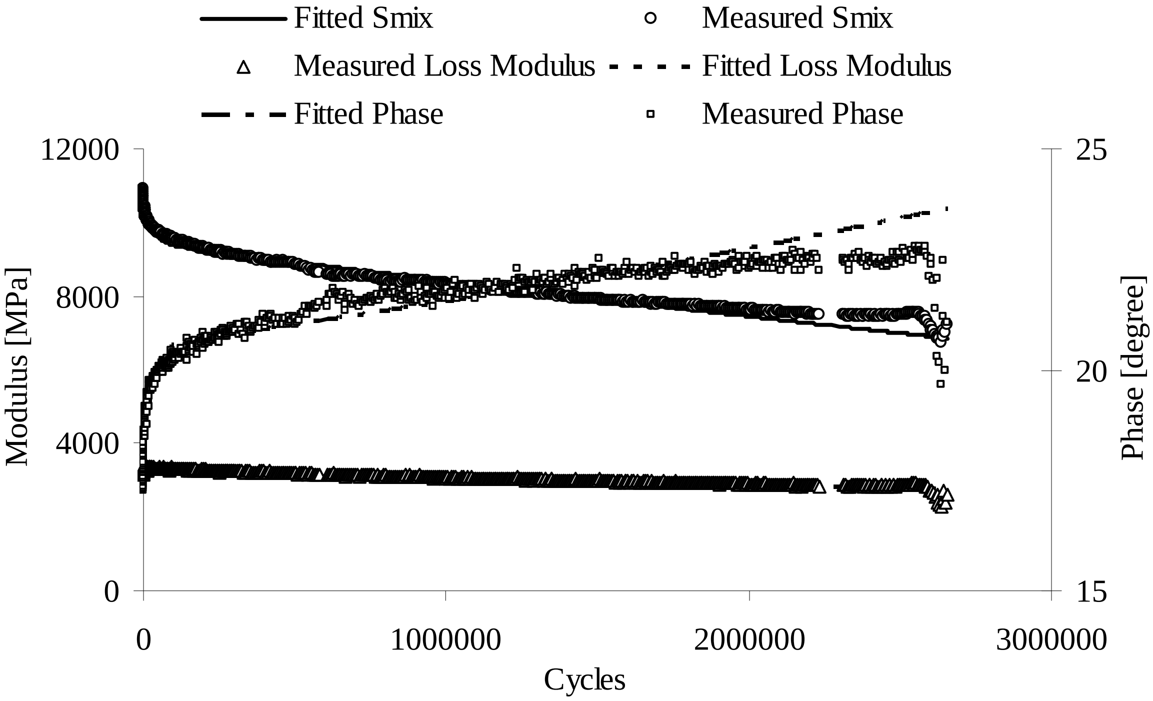

Figure 2: Measured and fitted evolutions of the complex stiffness modus for cylinder E13D80-14 using the PH parameters values of table 3 ($\delta\alpha_1^*= 0$ and $\beta^* = 52{,}000\ s^{-1}$).

4 MODIFIED PH MODEL

In figures 3 and 4 the values of table 3 are plotted as a function of the strain amplitude. The trends of the values for the permanent damage regression coefficients $\delta\gamma_1^*$ and $\delta\gamma_2^*$ in figure 3 indicate the existence of the so called endurance limit (equation 8). Here the endurance limit is taken equal to 74 μm/m. The best relation for $\gamma_1^*\delta$ is than: $0.240\ 10^6\ (\varepsilon - 74\ 10^{-6})$.

$$\delta\gamma_{1,2}^{*} = \gamma_{1,2}^{**}\left(\varepsilon - \varepsilon_{limit}\right)\ for\ \varepsilon > \varepsilon_{endurance}\ and\ nil\ for\ \varepsilon < \varepsilon_{endurance} \tag{8}$$

Below the endurance limit the stiffness modulus will still decrease in the start of a fatigue test but will also restore completely. The decrease is more than can be expected of the small increase in temperature (less than 1.5 degrees Celsius in a temperature cabinet with forced convection temperature control). The intercept of the trend line in figure 4 is taken equal to nil because for $\varepsilon = 0$ the PH parameters should be nil as well. Based on these calculations the parameters for the modified PH model which are used for the prediction of the evolution of the complex stiffness modulus in 4PB tests are presented in table 4. The only parameters which will be fitted for the comparison with the measured evolution in the 4PB tests are the initial loss modulus and storage modulus. Asphalt is not a very homogenous material and due to the natural variability the parameters might vary per specimen. Therefore in a second step some of the parameters mentioned in table 4 will also be varied in order to investigate whether another combination of parameters yields a better fit.

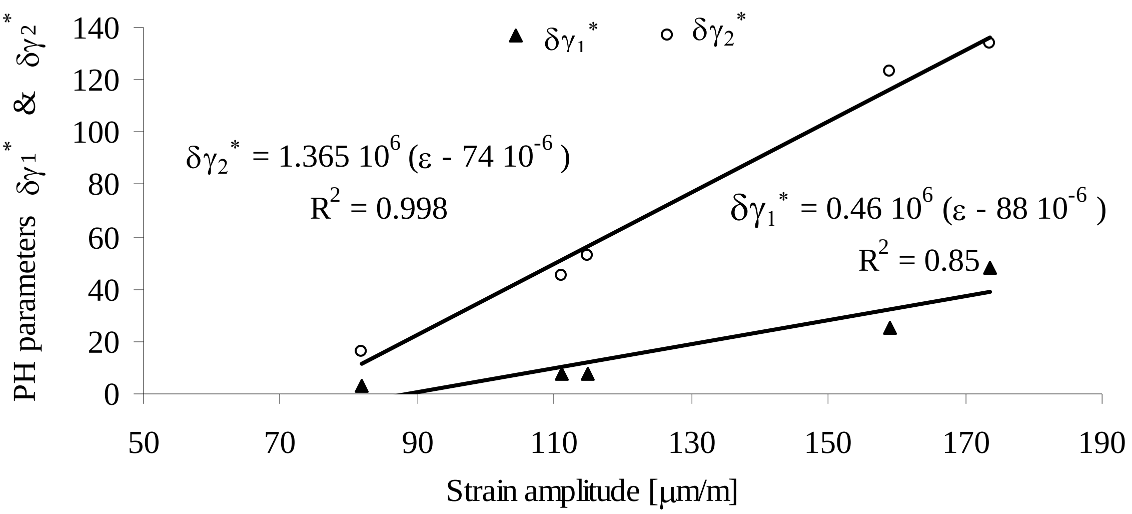

Figure 3: The PH parameters $\delta\gamma_1^*$ and $\delta\gamma_2^*$ as a function of the strain amplitude ε (table 3).

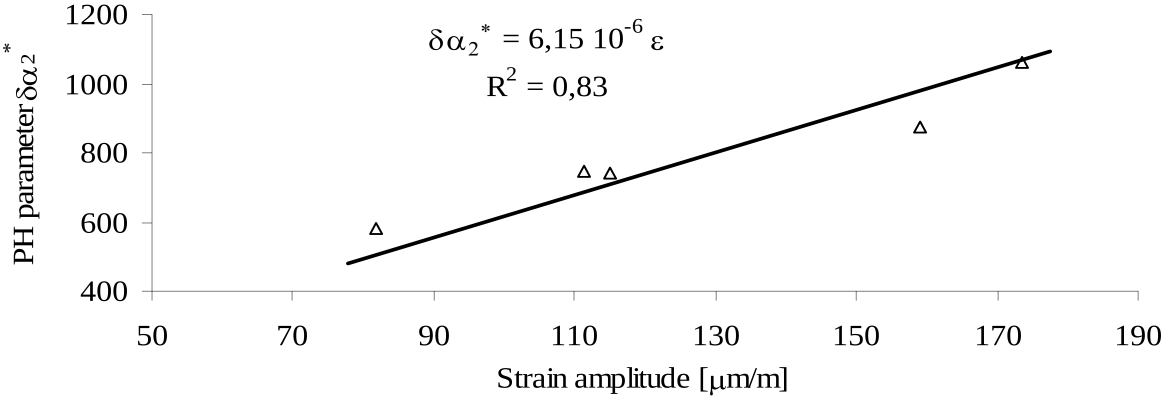

Figure 4: The PH parameter $\delta\alpha_2^*$ as a function of the strain amplitude ε (table 3).

Table 4: PH parameter values used for the prediction of stiffness evolution in 4PB tests.

$\delta\alpha_1^* = \alpha_1/(\pi f \varepsilon^2)$ [-/-]	$\equiv 0$	
$\delta\alpha_2^* = \alpha_2(\pi f \varepsilon^2)$ [-/-]	$= \delta\alpha_2^{**}\,\varepsilon$	$= 6.15\ 10^6\ \varepsilon$
$\delta\gamma_1^* = \gamma_1/(\pi f \varepsilon^2)$ [-/-]	$= \delta\gamma_1^{**}\,(\varepsilon - \varepsilon_{lim})$	$= 0.240\ 10^6\,(\varepsilon - \varepsilon_{lim})$
$\delta\gamma_2^* = \gamma_1/(\pi f \varepsilon^2)$ [-/-]	$= \delta\gamma_2^{**}\,(\varepsilon - \varepsilon_{lim})$	$= 1.365\ 10^6\,(\varepsilon - \varepsilon_{lim})$
$\beta^* = \beta/\varepsilon^2$ [s^{-1}]	= 52,000	
ε_{lim} [μm/μm]	$= 74\ 10^{-6}$	

5 FOUR POINT BENDING DATA

In the same RILEM project 12 prismatic beams ($450*50*50\ mm^3$) were tested in a 4PB device of which the outer span was 400 mm and the mid span was 130 mm. The beams were sawn from 4 slabs. Three strain levels were used in the tests at a frequency of 9.8 Hz and at a temperature of 20^0C. An overview is given in table 5 together with the fatigue life N1 (based on a change in the dissipated energy ratio; Hopman, 1989) and the traditional fatigue life $N_{f,50}$ (based on a 50% reduction of the initial stiffness modulus).

Table 5: 4PB beams used in the RILEM project.

Beam code	Strain ε [μm/m]	N1 [k cycles]	$N_{f,50}$ [k cycles]
0501	137	510	1200
0503	218	48	110
0504	177	170	340
A501	137	400	1000
A503	178	130	270
A504	217	42	90
A601	138	200	430
A602	219	90	200
A604	179	160	270
B601	218	120	200
B602	137	270	460
B603	177	120	215

6 NUMERICAL INTEGRATION OF THE PH MODEL

In contrast with the UPP test the 4PB test is not a homogenous test with respect to the strain distribution. The strain is maximal at the surface in the mid span and decreases linearly to nil at the neutral zone halfway through the beam. In the outer sections the strain decreases to nil at the outer supports. At this location the material will in principle not be damaged in the fatigue test. Therefore an integration of the PH model over the whole specimen has to be performed. This is done in a numerical way using Simpson's rule. One half of the outer section of the beam is divided in 10*10 = 100 "unit volumes". For the mid span the stiffness modulus distribution is equal to the one at the inner support. The weighted loss modulus F and weighted storage modulus G for the beam are calculated according to equation 9 because in the bending of the beam the product of the stiffness modulus and the bending moment is the relevant parameter.

$$\overline{F}\{t\} = \frac{\int_0^{1.5}\int_0^{1} F\{x,y,t\}\, y^2 dy\, dx}{\int_0^{1.5}\int_0^{1} y^2 dy\, dx} \quad \& \quad \overline{G}\{t\} = \frac{\int_0^{1.5}\int_0^{1} G\{x,y,t\}\, y^2 dy\, dx}{\int_0^{1.5}\int_0^{1} y^2 dy\, dx} \qquad (9)$$

7 APPLICATION OF THE MODIFIED PH MODEL ON THE RILEM 4PB DATA

The PH parameters in table 4 are used for a comparison of the evolutions according to the modified PH model with the measured evolutions. Only the initial loss modulus and storage modulus are fitted. The comparisons between the measured evolutions and the evolutions according to the modified PH model were surprisingly fair to good for all beams as shown in figure 5 for beam 0501. Probably due to the integration over the beam and the possible existence of an endurance limit, the 'information' in the evolution of the complex stiffness modulus for the beam is sometimes not enough for determining a unique solution. The fitting is carried out by first varying Fo and Go alone, next adding $\delta\alpha_2^*$ and β^* and at last $\delta\gamma_1^*$, $\delta\gamma_2^*$ and the endurance limit ε_{lim} are added in the fit. The result of the fitting for beam 0504 is presented in figure 6.

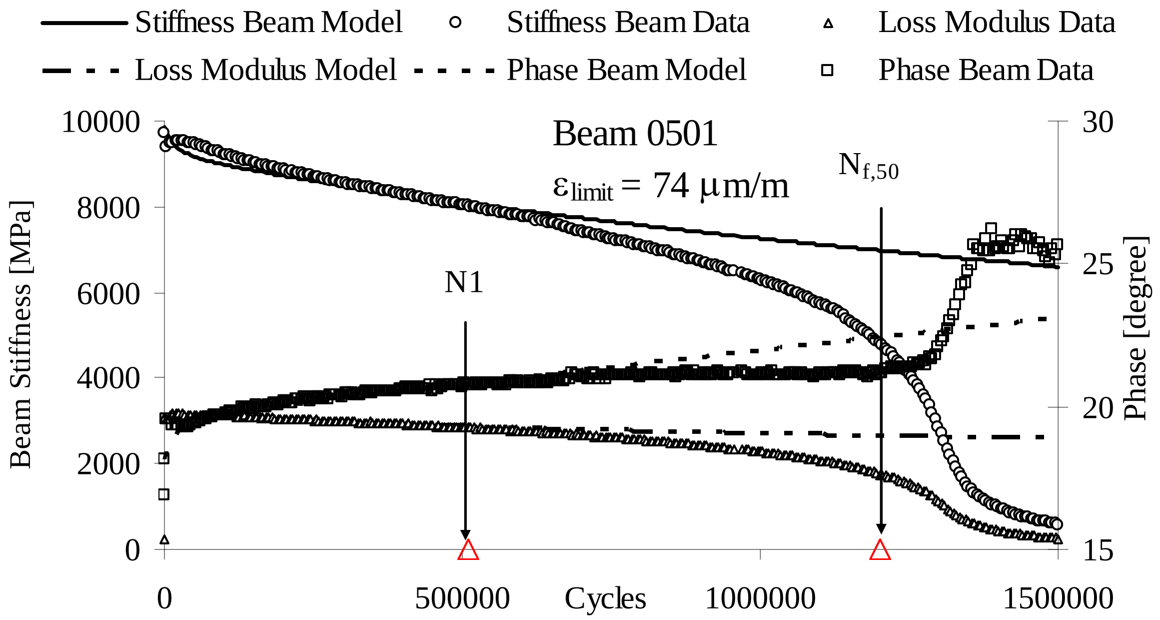

Figure 5: Comparison of the evolution for the complex stiffness of beam 0501 and the fitted evolution using the modified PH model with the parameter values of table 4.

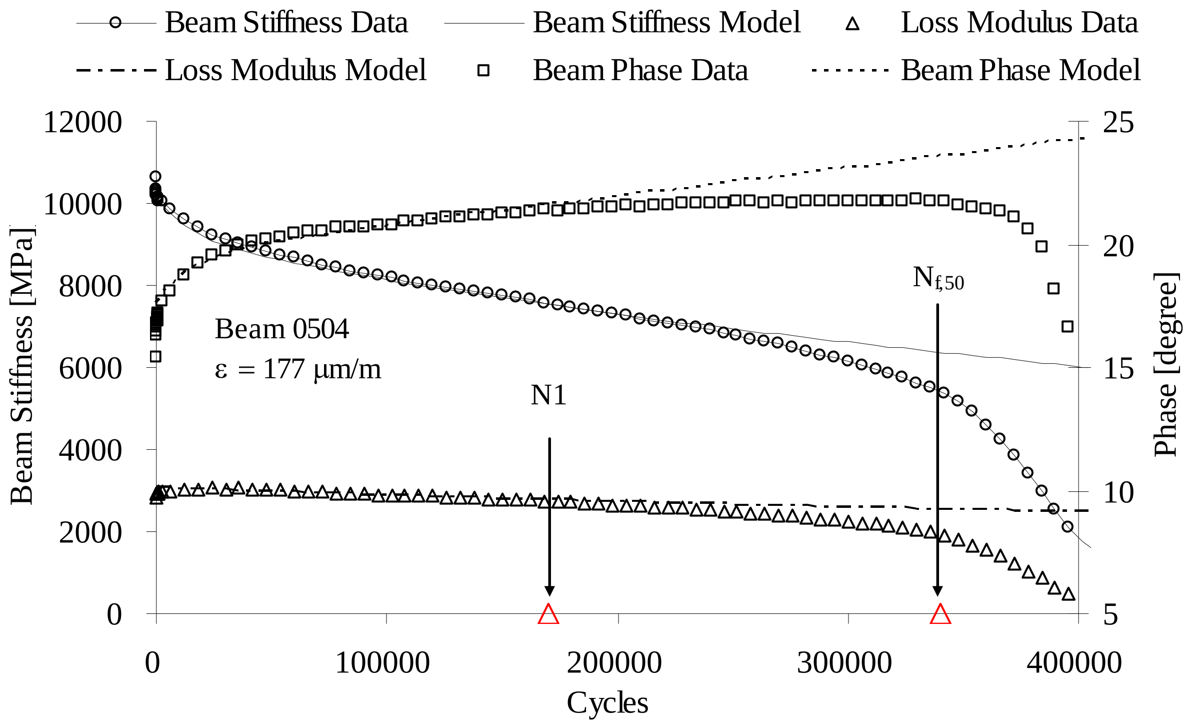

Figure 6: Comparison of the evolution for the complex stiffness of beam 0504 and the fitted evolution using the modified PH model varying all parameters of table 4.

The results for all beams following this protocol are given in table 6. To clarify the “lack” of information in the evolution of the complex beam stiffness for a good iteration a fit for beam A504 with complete other values for the parameters is given in figure 7. It is quite clear that this fit is also very good.

Table 6: Parameter values for the modified PH model in fitting the 4PB tests.

Beam	$\delta\alpha_2^*$ [10^6]	$\delta\gamma_1^{**}$ [10^6]	$\delta\gamma_2^{**}$ [10^6]	β^* [s^{-1}]	ε_{lim} [µm/m]
0501	6.15	0.240	1.36	52000	59
0503	7.88	0.361	1.55	82400	0
0504	6.15	0.240	1.36	41700	50
A501	6.15	0.327	1.35	49250	30
A503	6.15	0.240	1.36	49300	43
A504	6.15	0.560	2.22	76700	12
A601	6.15	0.291	1.36	58500	9
A602	6.15	0.318	1.35	141200	67
A604	6.15	0.316	1.35	54400	26
B601	6.15	0.314	1.35	114700	47
B602	6.15	0.309	1.35	63500	4
B603	6.15	0.297	1.35	70200	18

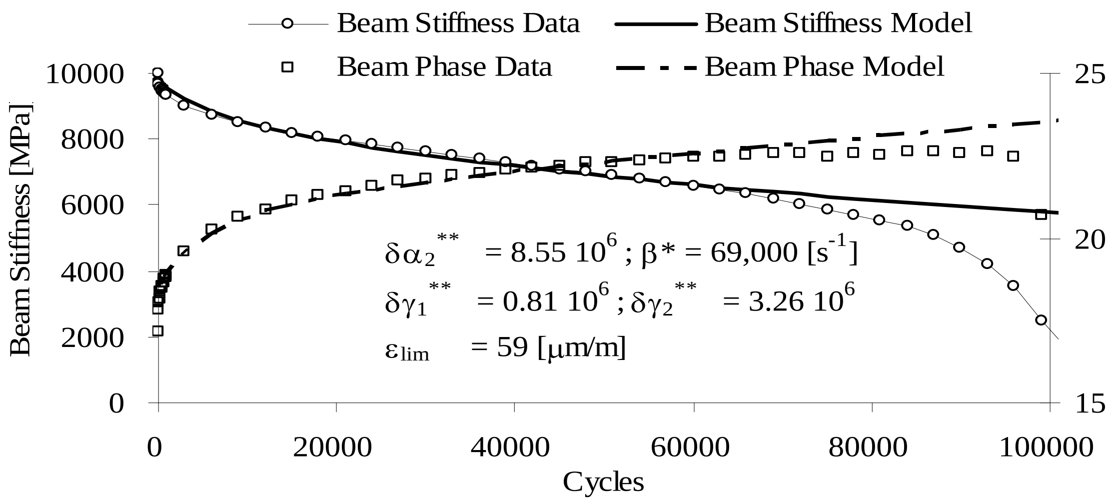

Figure 7: Comparison of measured data and the fitted modified PH model for beam A504 using different parameter values than given in table 4.

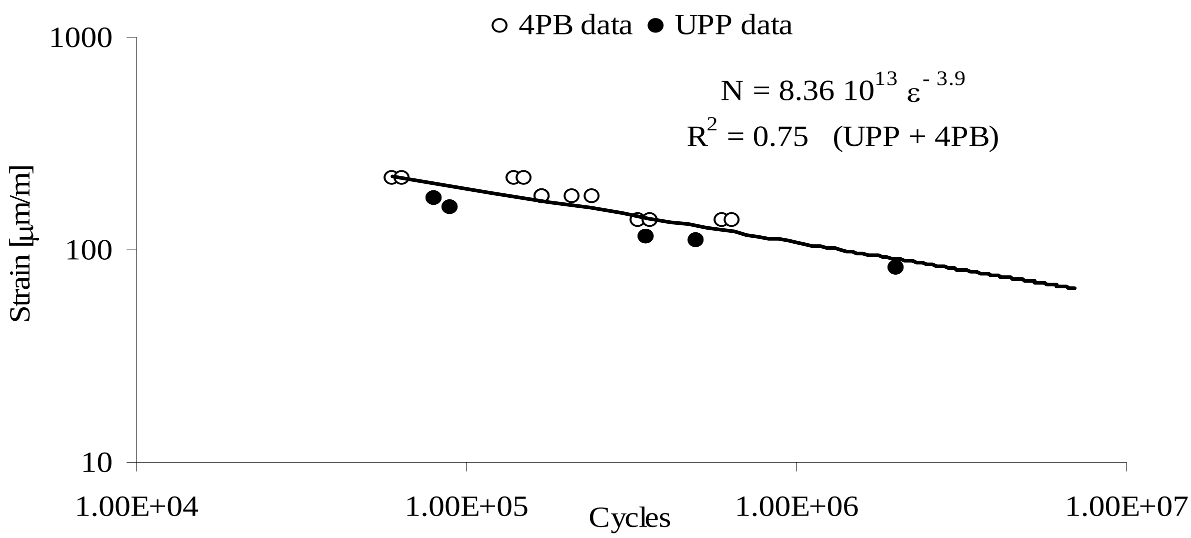

Figure 8: Comparison of fatigue lives N_{PH} determined in UPP and 4PB tests.

8 PROPOSED NEW FATIGUE LIFE DEFINITION

Next to the traditional definition $N_{f,\,50}$ for the fatigue life, the definition N1 based on a change in the dissipated energy per cycle is used in the interpretation of fatigue tests. However, both definitions fail when fatigue lives measured with different devices are compared. Given the very good fits using the modified PH model for the UPP and 4PB tests it is worthwhile to investigate if another definition might overcome this aspect. This fatigue life N_{PH} is defined as the moment when the measured data for the complex stiffness modulus starts to deviate from the evolution fitted with the modified PH model. Clearly the determination of this moment is not really objective but as shown in figure 8 the values of N_{PH} for the UPP and 4PB tests follow the same Wohler curve. Although the UPP fatigue lives are located at one side of the trend line this raises the confidence in the application of the PH model.

9 CONCLUSIONS

- The modified PH model describes the evolution of the complex stiffness modulus in a UPP test very well including the possible existence of an endurance limit.
- The material parameters of the modified PH model established in a UPP test can be used for describing the evolution of the weighted beam stiffness in a 4PB test. But good fits for the 4PB tests are also possible with different sets of material PH parameters.
- The numbers of cycles at which the comparable fits start to deviate from the measured data do not differ much. This number can be used as a fatigue life definition.
- The proposed fatigue life definition N_{PH} has potential to align results from UPP and 4PB tests. For this project it turns out that the value for N_{PH} is only a bit higher than N1.

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