Use of Fuzzy Analytic Hierarchy Process in Pavement Maintenance Planning

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ABSTRACT: In pavement management, prioritized resource allocation is imperative given the fact that there is rarely enough funding with highway agencies to address all the pavement sections. In view of the complexity involved in the decision making process, subjective evaluation based on expert's judgments is unavoidable. Analytic Hierarchy process (AHP) has been applied by researchers to solve pavement resource allocation problems in a multiple criteria environment. However, it has been reasoned that uncertainties and vagueness in human perceptions and judgments have to be considered. A fuzzy AHP (FAHP) and fuzzy arithmetic methodology is proposed in this study for maintenance prioritization in pavement maintenance planning. The goal is to identify an approach that can reflect the intention of highway agency and engineers more closely during decision making process. Given the fact that there is no analytical technique or tool that permits one to make comparison on a theoretical basis, numerical examples are employed to evaluate the relative merits of different methods in this paper. The study concludes that the proposed fuzzy arithmetic method is suitable for the pavement maintenance prioritization process, based on its ability to prioritize pavement maintenance activities in good agreement with the Direct Assessment Method, and its operational advantage in evaluating a large number of maintenance activities.

KEYWORDS: Pavement maintenance prioritization, fuzzy AHP, direct assessment method, fuzzy logic, priority ranking.

1 INTRODUCTION

In pavement management, the purpose of maintenance is to execute protective and repair measures in order to slow down the pavement deterioration process, thereby extending the useful life of a pavement. The efficacy of pavement maintenance is highly increased, if action is taken at an appropriate time in a preplanned manner. Practically all pavement management system consists of priority models to prioritize pavement projects or pavement maintenance activities. These models range from simple ranking to complex optimization models. The quality of prioritization process can directly influence the effectiveness of available resources that are, in most cases, the primary judgment of the decision maker. A widely adopted practice by highway agencies is to rank all the pavement maintenance projects or activities exhaustively using experts' judgment, and execute the top ranking activities until all funds are expended (Sharaf, 1993).

Pavement maintenance priority is often represented by empirical mathematical indices (Fawcett, 2001, Broten, 1996, Barros, 1991), which often do not have a clear physical meaning, and could not accurately and effectively convey the priority assessment or intention of highway agencies and engineers. Furthermore, there is a paradox inherent in empirical indices such as, on one hand it has to be comprehensive to present an accurate picture in the mind of a decision maker, and on the other hand merging excessive information may render the index meaningless as too many different things are being measured at the same time.

Furthermore, it is often difficult to map qualitative preferences to point estimates, and a degree of uncertainty will be associated with some or all of the judgments. In an attempt to overcome the above mentioned limitations, this study explores the use of the fuzzy Analytic Hierarchy Process (FAHP) (Cheng, 1997, Chang, 1996, Buckley, 1985) for prioritization of pavement maintenance activities. According to cognitive psychology, humans are poor at assimilating large quantities of information on problems therefore fuzzy AHP based on a principle of pairwise comparison to capture decision makers' preferences provides a useful tool in pavement maintenance prioritization. It is believed that the proposed approach will ensure a more convincing decision-making by effectively representing and processing ambiguities involved in the assessment data. The results are assessed by comparing with the priority assessments obtained from a Direct Assessment Method (Fwa et al., 1989) in which the raters make the evaluation by comparing all the maintenance activities together directly.

2 CONCEPT OF FUZZY ANALYTIC HIERARCHY PROCESS

The fuzzy set theory (Zadeh, 1965) permits the assessment of the membership of elements with respect to a set, represented by $\mu \rightarrow [0, 1]$, in contrast to classical set theory. The fuzzy Analytic Hierarchy Process is an extension of widely employed Analytic Hierachy Process (AHP) technique (Saaty, 1980, 1990, 1994, 1995) for multicriteria decision making. Fuzzy AHP involves the following phases: (a) structuring of a hierarchy, (b) prioritization based on fuzzy pairwise comparison, (c) checking for consistency of the preference judgments, (d) synthesis of pairwise priorities and (e) defuzzification of priorities.

A hierarchy is an effective way of organizing and representing a complex problem. According to Saaty (1980), a hierarchy is "an abstraction of the structure of a system to study the functional interactions of its components and their impacts on the entire system." It consists of an overall goal at the top or first level, a set of alternatives, at the bottom or last level, for reaching the goal, and a set of criteria, at mid-level, that relate the alternatives to the goal.

The next phase is pairwise comparison of criteria using fuzzy numbers. A simplistic approach is to construct a triangular fuzzy number Ã by using most probable value and range to represent a judgment. According to Kaufmann (1991), a fuzzy number Ã is a triangular fuzzy number (TFN) if its membership function $\mu_{\tilde{A}}(x) : \mathbb{R} \rightarrow [0, 1]$ equates to the following,

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - l/m - l), & l \le x \le m \\ (u - x/u - m), & m \le x \le u \\ 0, & otherwise \end{cases} \tag{1}$$

where l and u represent the lower and upper bounds of the fuzzy number Ã respectively, while m is the median value as shown in Figure 1. Hence, a TFN is denoted by Ã= (l, m, u), and fuzzy operation on triangular numbers are described as follows,

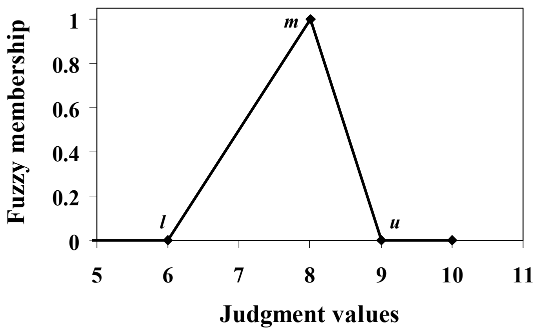

Figure 1: Fuzzy triangular number (TFN) Ã = (6, 8, 9)

Fuzzy number addition $\oplus$: $\tilde{A}_1 \oplus \tilde{A}_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$ (2)

Fuzzy number subtraction $\ominus$: $\tilde{A}_1 \ominus \tilde{A}_2 = (l_1, m_1, u_1) \ominus (l_2, m_2, u_2) = (l_1 - l_2, m_1 - m_2, u_1 - u_2)$ (3)

Fuzzy number multiplication $\otimes$: $\tilde{A}_1 \otimes \tilde{A}_2 = (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) \cong (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2)$ for $l_i > 0$, $m_i > 0$, $u_i > 0$ (4)

Fuzzy number division $\oslash$: $\tilde{A}_1 \oslash \tilde{A}_2 = (l_1, m_1, u_1) \oslash (l_2, m_2, u_2) = (l_1 / l_2, m_1 / m_2, u_1 / u_2)$ for $l_i > 0$, $m_i > 0$, $u_i > 0$ (5)

Fuzzy number logarithm: $\log_n(\tilde{A}) \cong (\log_n l, \log_n m_1, \log_n u_1)$ n is base (6)

Fuzzy number reciprocal: $(\tilde{A})^{-1} \cong (l, m, u)^{-1} \cong (1/u, 1/m, 1/l)$ for l, m, $u > 0$ (7)

Findings from psychological studies by Miller (1956) have shown that individuals are unable to effectively apply a rating scale of more than seven (plus or minus two) points. The preference judgment made by the decision maker is represented by triangular fuzzy number whose values range from $\tilde{1}$ to $\tilde{9}$ as shown in Table 1.

Table 1: Memebrship function of linguistic scale

Fuzzy number	Membership function	Linguistic scales
$\tilde{1}$	(1, 1, 3)	Equally important
$\tilde{3}$	(1, 3, 5)	Weakly more important
$\tilde{5}$	(3, 5, 7)	Strongly more important
$\tilde{7}$	(5, 7, 9)	Very strongly more important
$\tilde{9}$	(7, 9, 9)	Absolutely more important

The outcome of each set of pairwise comparisons is expressed as a fuzzy positive reciprocal matrix $\tilde{A} = (\tilde{a}_{ij})$ such that $\tilde{a}_{ii} = 1$ and $\tilde{a}_{ij} = 1/\tilde{a}_{ji}$ for all $i, j \leq n$, where n denotes the number of alternatives being compared within one set of pairwise comparisons, $\tilde{a}_{ij}$ denotes the importance of alternative i over alternative j, and $\tilde{a}_{ji}$ denotes the importance of alternative j over alternative i, and $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$.

AHP allows for 10 percent inconsistency in human judgments (Saaty, 1980), hence the same principle is applied in fuzzy AHP. To check for consistency in judgments of a decision maker, Saaty (1994) defined the consistency ratio *CR* which is a comparison between Consistency Index *CI* and Random Consistency Index *RI* as follows,

$$CR = \frac{CI}{RI} \tag{8}$$

where *CI* is given by

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{9}$$

where largest eigenvalue called the principal eigenvalue λ_{max} of the matrix *A*, n is the size of the matrix. *RI* is obtained by computing the *CI* value for randomly generated matrices. A matrix is considered consistent, only if $CR \le 0.1$ (1990). The consistency of the judgments is evaluated by means of the median value in the triangular fuzzy number.

The geometric mean technique (Aczel et al., 1983) can be extended to the fuzzy positive reciprocal matrices Ã in order to determine weights $\tilde{w}_i$ (Buckley, 1984). Given a positive reciprocal matrix Ã, the geometric mean of each alternative is given by,

$$\tilde{r}_i = \left(\prod_{i=1}^{n} \tilde{a}_{ij} \right)^{1/n} \tag{10}$$

The weight w_i of criterion or alternative is given by,

$$\tilde{w}_i = \tilde{r}_i \otimes \left(\sum_{i=1}^{n} \tilde{r}_i \right)^{-1} \tag{11}$$

It is established for each criterion, sub-criterion, as well as the alternatives under each sub-criterion. The overall fuzzy weight of alternatives is computed as follows,

$$\tilde{V}_i = \sum_{j=1}^{n} \tilde{w}_j \, \tilde{r}_{ij} \tag{12}$$

where $\tilde{V}_i$ = overall fuzzy weight of alternative *i,* $\tilde{w}_j$ = fuzzy weight assigned to criterion *j*, and $\tilde{r}_{ij}$ = fuzzy weight of alternative *i* given criterion *j*.

The overall fuzzy number $\tilde{V}_i$ needs to be defuzzified to generate a crisp value which results in establishing the final priority weights and ranks of alternatives. The defuzzification is realized by employing fuzzy mean and spread method (Lee et al., 1988) as follows,

$$x(\tilde{V}_i) = (l + m + u)/3 \tag{13}$$

where $\tilde{V}_i$ is assumed to follow uniform distribution with a mean $x(\tilde{V}_i)$.
The fuzzy numbers are ranked as per $x(\tilde{V}_i)$ to determine the priority ranking of each alternative. The aforementioned procedure requires *n(n-1)/2* pairwise comparisons for *n* number of items, hence for $n = 27$, the number of comparisons would be 351.

3 CONCEPT OF FUZZY ARITHMETIC METHOD

In order to reduce the number of pairwise comparisons, a variant of AHP using fuzzy arithmetic operations is employed. The basic methodology remains the same as fuzzy AHP approach; however the main difference occurs at the last level during the evaluation of alternatives. The alternatives are each assigned a degree of intensity under each covering criterion. By doing so, the number of pairwise comparisons involved in the fuzzy AHP computation is reduced substantially. It should be noted that intensities are still required to be compared pairwise.

4 METHODOLOGY OF STUDY

The objective of this study is to evaluate the practical suitability and effectiveness of the two fuzzy methods in establishing the priorities of maintenance activities for developing a pavement maintenance program. Given the fact that there does not exist any analytical technique or tool that allows comparison or validates the proposed procedures on theoretical basis, numerical examples are used to assess the relative merits of different methods. It is considered that the "real" set of priorities must be established by the engineers and highway agency concerned, hence the "Direct Assessment Method" using a card approach as employed by Fwa et al. (1989) is adopted.

The card approach represents a convenient approach to rank alternatives on a scale ranging from 1 to 100 such that an expert is presented with all the alternatives together, and is given enough time to rank the alternatives, and make adjustments until acceptable satisfaction is achieved. The evaluator was first asked to place the cards in rank order according to the respective urgency of the need to perform the maintenance activities. Next, they are required to move the cards into relative positions above or below each other along a linear scale of 1 to 100. The end results of this survey will provide the priority rating of each maintenance activity ranked on a scale of 1 to 100.

5 ILLUSTRATIVE EXAMPLE

5.1 Problem Formulation

For illustration purpose, three road functional classes, three distress types, and three level of distress severity are considered. This results in 27 possible combinations of maintenance treatments. The three road functional classes include expressway, arterial and access road. The three types of distresses include pothole, rutting and cracking, and the three levels of distress severity associated with each of these distresses are high, moderate and low.

For the sake of simplicity in presentation, only 27 pavement sections are considered, each with a different maintenance activity corresponding to one of the 27 possible combinations of maintenance treatments as shown in Figure 2.

5.2 Application of Fuzzy Analytic Hierarchy Process

The primary step is to decompose a problem into individual independent elements by developing a hierarchy. Figure 2 shows the hierarchy structure used for the fuzzy AHP analysis of the example problem. The overall objective is to prioritize pavement sections for the execution of pavement maintenance activities, and is placed at the top level in the hierarchy.

For the considered problem, factors influencing maintenance activities are translated as criteria for the ranking of alternatives. Class of pavement is selected as the main criterion, directly influencing the overall objective, and is placed at second level in the hierarchical structure. Type of distress is selected as the sub-criterion to the main criterion, and is placed at the third level. Level of severity of the distress type, sub-criterion to sub-criterion, is placed at the fourth level, while twenty seven pavement sections are identified as the candidates for the execution of pavement maintenance activities, and are placed at the bottom or the fifth level in the hierarchy.

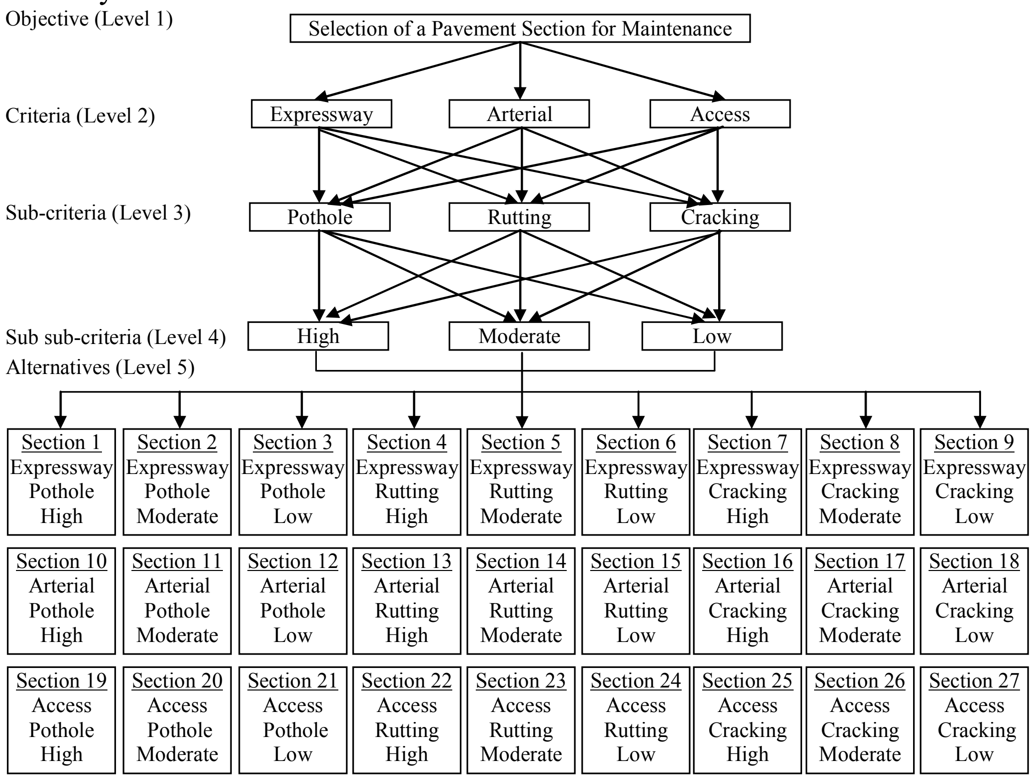

Figure 2: Hierarchy structure for AHP analysis of example problem

Prioritization involves pairwise comparisons between elements at the same level in the hierarchical structure by the rater. Five pavement engineers were asked to provide priority ratings for the 27 pavement sections using the fuzzy methods and the Direct Assessment Method. For the fuzzy AHP and fuzzy arithmetic methods, all pairwise comparison matrices were checked for consistency using the consistency ratio defined by Eq. (8). Some of the matrices were found to be inconsistent, and the experts concerned were requested to revise their judgments.

6 ANALYSIS OF PRIORITY RATING SCORES AND PRIORITY RANKINGS

The suitability and effectiveness of the fuzzy AHP and fuzzy arithmetic procedures are assessed using the following analysis:

(a) Assessment of priority rating scores by comparing the statistical correlations with scores obtained using the Direct Assessment Method;
(b) Assessment of priority rankings by comparing the rank correlations with rankings obtained using the Direct Assessment Method;
(c) Hypothesis testing of consistency of priority rankings with the Direct Assessment Method

6.1 Assessment of Priority Rating Scores

Figure 3 presents the plots of priority rating scores obtained using the two fuzzy methods against those by the Direct Assessment Method. Also indicated in the two figures are the

Pearson correlation coefficients (Neter et al., 1990). Pearson correlation coefficient *r* represents the strength of linear relationship between the Direct Assessment Method and the fuzzy methods.

It is observed from Figure 3 that the priority rating established using the two fuzzy methods were quite different from the corresponding values by the Direct Assessment Method. These discrepancies are believed to be reflective of the differences in the basic approach of survey adopted by the fuzzy methods and the Direct Assessment Method, and the different rating scales employed by them. The primary difference between the Direct Assessment Method and fuzzy methods is that in the latter the rater is oblivious of the final outcome while rating alternatives, thus the priorities generated are free from bias, and solely depend on the preference judgments made by the decision maker during pairwise comparison.

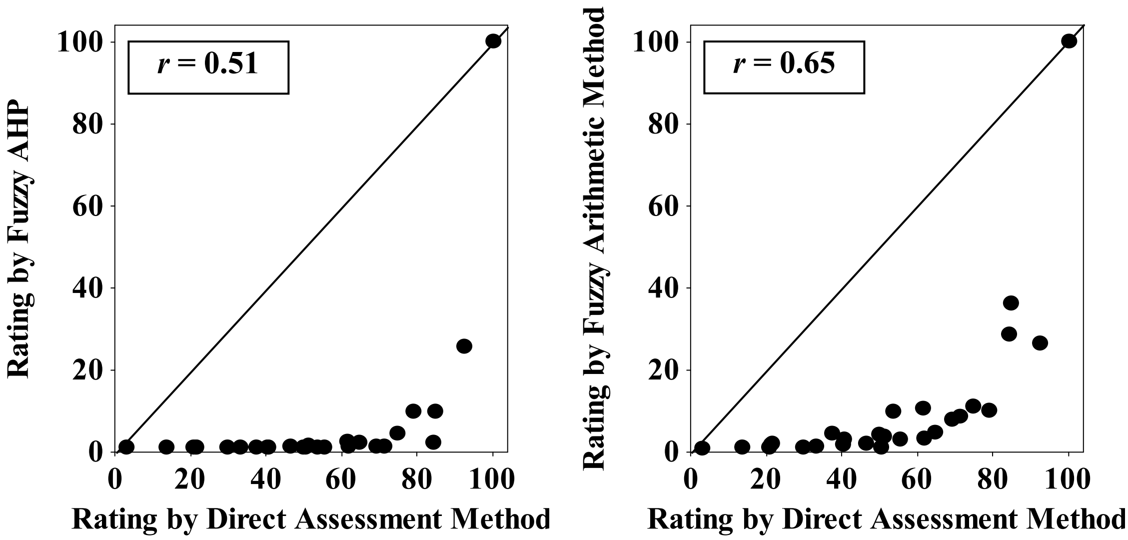

Figure 3: Correlations between the priority ratings obtained using Direct Assessment Method, fuzzy AHP and fuzzy arithmetic methods

6.2 Assessment of Priority Rankings

Figure 4 presents the plots of priority rankings established by employing the two fuzzy methods against those by the Direct Assessment Method. As can be seen from the Figure there is a strong positive correlation between each of the fuzzy methods and the Direct Assessment Method. The fuzzy AHP has the highest correlation of 0.93, while the fuzzy arithmetic method has the correlation of 0.83. These results suggest that the two fuzzy methods were able to generate priority rankings of pavement maintenance activities in good agreement with the Direct Assessment Method, however fuzzy AHP tends to show higher correlation than fuzzy arithmetic method probably on account of no pairwise comparison involved in the latter while comparing alternatives at the last level in the hierarchy in contrast to the Direct Assessment Method and the fuzzy AHP. Statistical testing of the degree of this agreement is delineated in the subsequent section.

6.2.1 Statistical Testing of Rank Correlation

The strength of association of each of the fuzzy methods with the Direct Assessment Method is evaluated using statistical hypothesis testing based on the non-parametric Spearman rank correlation test (Lehmann et al., 1998). The test parameter is the Spearman rank correlation coefficient ρ defined as follows,

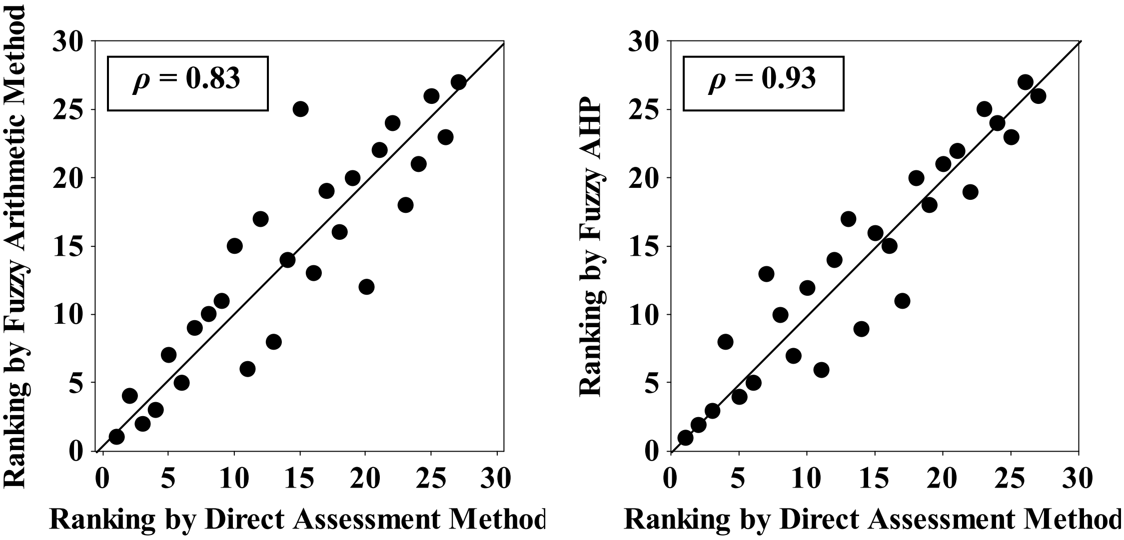

Figure 4: Correlations between the priority rankings obtained using Direct Assessment Method, fuzzy AHP and fuzzy arithmetic methods

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2 - 1)} \tag{14}$$

where d_i = difference between the ranks of pavement section i by the Direct Assessment Method and the fuzzy method being evaluated, and all other variables as defined in Eq. (14). As correlation coefficient exceeding 0.6 indicates very strong degree of correlation (Franzblau, 1958), the test was performed with the null hypothesis H_0: $\rho \leq 0.6$, against the alternative hypothesis H_1: $\rho > 0.6$. The results of the hypothesis tests are summarized in Table 3.

Table 3: Spearman's rank correlation coefficient and Student's t-test

Statistic	Fuzzy AHP	Fuzzy Arithmetic
Observations	27	27
α	0.05	0.05
Correlation	0.93	0.83
Student's T-test (t_{n-2} *for* $n > 10$)	11.95	7.21
Critical 1-sided T-value $(t_{\alpha,n-2})$	1.708	1.708
Conclusion	Accept H_1: $\rho > 0.6$	Accept H_1: $\rho > 0.6$

6.3 Applicability of Fuzzy AHP and Fuzzy Arithmetic Procedure

As far as suitability of each method for priority assessment is concerned, it is appropriate to consider the number of pairwise comparisons required to be made in order to arrive at the final priority assessment. The Direct Assessment Method is intuitively the method a person would use in making priority assessment. In theory, to rank and rate n number of items, the Direct Assessment Method would involve $n(n-1)/2$ number of comparisons. For the example problem analyzed in the preceding section, the number of comparisons would be 351.

For the same example, the numbers of comparisons required were 129 and 21 for the fuzzy AHP and the fuzzy arithmetic method respectively.

Table 4: Number of comparisons required for each method

Description and Size of Problem	Number of Judgments Required		
	Direct Assessment Method	Fuzzy AHP	Fuzzy Arithmetic
Three levels of criteria = (Road functional class; Distress type; Distress severity) Total number of maintenance alternatives = 27	351	129	21
Five levels of criteria = (Road functional class; Distress type; Distress severity; Traffic loading; Climatic condition) Total number of maintenance alternatives = 243	29403	9759	39

Note: Each criterion has three sub-criterions

From the standpoint of practicality, the number of pavement maintenance alternatives involved would be much more than $n = 27$ in the actual road network. The extent of the problem can also be increased if more factors are added in the priority assessment process. For instance, besides the three factors considered in the example problem (i.e. road function class, distress type, and distress severity level), more factors such as the level of traffic loading and climatic condition can be included. Taking the example problem as an illustration, by adding these two additional factors, the number of comparisons needed would be 29403, 9759 and 39 for the Direct Assessment Method, the fuzzy AHP and the fuzzy arithmetic method respectively, as shown in Table 4.

Evidently, the fuzzy arithmetic procedure is considered suitable and manageable in terms of performing priority assessment for network level planning and programming of pavement maintenance activities. However, fuzzy AHP is also suitable in terms of its strong agreement with the Direct Assessment Method.

7 CONCLUSIONS

Fuzzy AHP and fuzzy arithmetic method have been evaluated for their suitability and efficacy in the priority assessment of pavement maintenance activities. The evaluation was performed with reference to the Direct Assessment Method and based on that the following conclusions can be elicited:

(a) Strong association is evident between the Direct Assessment Method and the two fuzzy methods adopted in this study; however the fuzzy AHP methodology showed a significant degree of correlation.

(b) The fuzzy arithmetic method can rank large number of alternatives as opposed to the Direct Assessment Method. Furthermore, the former renders the priorities of alternative maintenance activities in good agreement with the latter. The strong correlation in rankings was confirmed through statistical hypothesis testing performed at a confidence level of 95%.

(c) Based on fuzzy arithmetic methods' operational advantage in handling a large number of items to be evaluated, and its ability to generate priority assessment in good agreement with the Direct Assessment Method, it is considered to be the preferred method for use in pavement maintenance prioritization.

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