Resilient Modulus Estimation for Granular Materials

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ABSTRACT

One of the important parameter in pavement design is the resilient modulus, which describes the materials mechanical behaviour and allows dimensioning the pavement multilayer system. The modulus and the thickness of each pavement layer allow analyzing the structures with stresses and strains distribution under wheel loads. However, since the resilient modulus determination is complex and the necessary equipment costly, the use of precise and adapted estimation methods is suitable for the implementation of mechanistic-empirical pavement design. This research objective is to develop an estimation model for the resilient modulus of typical Canadian granular materials. The resilient modulus is influenced by numerous factors. The proposed estimation approach uses explanatory variables linked with the water content, gradation and density, to determine appropriate *c1s* and *c2s* values for the saturated state in the linear constitutive model. The model was developed with 25 granular materials sampled at various locations in Canada.

KEYWORDS: Resilient modulus, Granular materials, Mechanistic-empirical, Pavements, Estimation.

1. INTRODUCTION

Flexible pavement design is based on dimensioning multilayered system which layer thicknesses vary depending on the subgrade bearing capacity. The use of the resilient modulus (M_R) for pavement design was, amongst others, suggested by the AASHTO (1986, 1993) in the late eighties to replace bearing capacity parameters such as CBR, Rvalue and SSV. Soils and unbound granular materials M_R is equivalent to an elastic modulus used with elasticity theory. It is an essential parameter for dimensioning, analyzing and designing pavement structures. In combination with the layer thickness, it can be used to define a multilayered system for which stresses and strains under wheel load can be analyzed. The M_R is expressed as the ratio of the deviatoric stress σ_d to the resilient strain ε_R as expressed in

$$M_R = \frac{\sigma_d}{\varepsilon_R} \tag{1}$$

for which the resilient modulus is typically expressed in MPa. The M_R value can be determined from laboratory tests, indirectly from typical values of regression constitutive laws parameters or indirectly from an estimation based on other test results. In this study, results obtained from direct measurements of resilient modulus through repeated load triaxial tests performed according to LC 22-400 standard (MTQ 2007) were used. The objective of the study is to analyze the available M_R database and to develop a prediction model based on the unbound granular materials physical and state properties.

2. FACTORS INFLUENCING M_R

It is now well recognized that the stress state is the parameter having the more pronounced effect on the resilient behaviour of soils and aggregates. As stated by Lekarp et al. (2000), the M_R tends to increase with an increase of confining pressure and total stress. Barksdale and Itani (1989) also reported an important effect of the density. Their results suggested an important M_R increase with a density increase at low stress, while this effect seems less pronounced at higher stress level. In addition, as stated by Zaman et al. (1994), the aggregate type as also an important effect on the resilient behaviour. The water content effect was studied by Thompson (1989), who reported a decrease of the resilient modulus with an increase of the saturation degree. The water content effect was analyzed in terms of matric suction by Doucet and Doré (2004). They developed a model describing the M_R as a function of matric suction based on a study performed on numerous base and subbase granular materials used for the C-LTPP project (Canadian Long Term Pavement Performance). In this model, the M_R (kPa) is expressed by

$$M_R = -1060\theta - 8700\left(u_a - u_w\right) + 57000 \tag{2}$$

in which θ (kPa) is the sum of the principal stresses and $(u_a\text{-}u_w)$ (kPa) is the matric suction.

3. RESILIENT MODULUS MEASUREMENT

3.1. Direct measurement according to LC-22-400 standard

A test method was developed by the Ministry of Transportation of Quebec (MTQ 2007) to characterize the non linear resilient modulus according to the stress state and the water content. This method is mostly based on the AASHTO T307-99 standard (AASHTO 2003). The main differences are that the sample is characterized at three water contents, the axial strain measurements are performed on sample (two axial gages 180° apart on the central 200 mm) and the conditioning cycle last 10 000 cycles. The three water contents used for resilient modulus characterization are: initial water content (set to 2% above the aggregate absorption value), saturated water content and drained water content (drainage under gravitational forces). This test method is performed on materials scalped on the 31.5 mm sieve, having a maximum plasticity index of 10% and having a maximum of 20% of fine particles. The 150 mm diameter and 300 mm ± 10 mm height samples are compacted in seven layers using a vibrating compaction hammer inside a stainless steel mould. After 10000 load cycles of conditioning under a confining (σ_3) and deviatoric (σ_d) stresses of 105 kPa, the M_R is characterized under the stress conditions presented in Table 1.

Table 1. Stress states for the resilient modulus characterization (MTQ 2007)

σ_3 (kPa)	σ_d (kPa)	σ_{do}* (kPa)	σ_{dr}* (kPa)	θ (kPa)
20	20	2	18	80
	40	4	36	100
	60	6	54	120
35	35	3	32	140
	70	7	63	175
	105	11	94	210
70	70	7	63	280
	140	14	126	350
	210	21	189	420
105	70	7	63	385
	105	11	94	420
	210	21	189	525
140	105	11	94	525
	140	14	126	560
	280	28	252	700

*: σ_{do}= 10% of σ_d static, σ_{dr} = 90% of σ_d cyclic

3.2. Indirect measurement

Indirect measurement of M_R is based, amongst others, on other bearing capacity index such as CBR and Rvalue. Similarly, an estimation of the resilient modulus can be obtained from typical k_1 and k_2 regression values, which are to be used with the well-known K-θ constitutive law. This law is generally used in Quebec (Canada) and describes the resilient modulus non linearity according to total stress. It is expressed by

$$M_R = k_1 P_a \left(\frac{\theta}{P_a} \right)^{k2} \tag{3}$$

in which P_a is the atmospheric pressure (kPa), θ the total stress (kPa) and k_1-k_2 are regression parameters. Uzan (1985) constitutive law can also be used when typical regression parameters are known. Table 2 summarizes typical values of k_1 and k_2 (Robert et al. 2002) to be used with the K-θ model for pavement granular materials. These typical values were obtained from resilient modulus tests performed on various types of granular materials for several aggregate sources. As it can be observed, a significant variability is found for k_1 and k_2. Robert and al. (2002) suggested that k_2 is less variable than k_1 and that it represents the material stresses sensitivity and a cohesion within the granular assembly.

Table 2. Typical values for k_1 and k_2 used in Quebec for the K-θ model

Material	k_1 (kPa)	k_2
Subbase sand	7 000 – 10 000	0.480 – 0.580
Crushed base	8 000 – 15 000	0.550 – 0.650
Partly crushed base	11 000 – 16 000	0.500 – 0.600
Recycled crushed concrete base	23 000 – 30 000	0.450 – 0.550
0-40 mm and 0-56 mm	5 000 – 12 000	0.480 – 0.660

3.3. Estimation models

Several authors suggested models to estimate the resilient modulus of soils and aggregates. Jones and Witczak (1972) defined a model that considers the saturation degree. Thompson and Robnett (1979) developed a resilient modulus estimation model for Illinois fine-grained soils that includes the silt and clay contents, the soil classification and the plasticity index. Several other similar studies were performed in various American states (Carmichael and Steward 1985, Elliot et coll. 1988, Drumm et coll. 1990, Farrar et Turner 1991, Hudson et coll. 1994, Li et Seling 1994, Pezo et Hudson 1994, Berg et coll. 1996). One of the more recent approaches was presented by Rahim and George (2005), who proposed resilient modulus estimation models for coarse and fine-grained Mississippi subgrade soils. In addition, Gupta et al. (2007) suggested an approach that considers the matric suction to include the effect of water content.

4. MODEL DEVELOPMENT

In this study, a resilient modulus database of 25 coarse-grained unbound granular materials used as pavement base and subbase was gathered from previous research projects (Bilodeau 2009, Doucet and Doré 2004). All the tests were performed at the Ministry of Transportation of Quebec laboratory using the same equipment and procedures (LC 22-400). Table 3 summarizes the materials included in the resilient modulus database, the material and aggregate type, as well as the soil classification according (USCS).

Table 3. Data source, material type, aggregate type and classification

Data source	Material	Aggregate type*	Classification (USCS)
Bilodeau (2009)	Base (0-20 mm)	Granitic gneiss Limestone Basalt	SW-SM, GW
Doucet and Doré (2004)	Base (0-20 mm) Subbase (0-112 mm)	Various	SP-SM, SP-SC, GW-GC, SM

*Crushed stone and crushed gravel

In order to precisely estimate the resilient modulus for Canadian unbound aggregate, a preliminary analysis was performed to identify the adequate constitutive law to use for the model development. The Uzan model, the K-θ model and a linear model were considered. The preliminary results suggested that the linear model allowed obtaining the best predictive capacity. This model was also used in the study of Bilodeau (2009) and Doucet and Doré (2004). The linear model is expressed

$$M_R\left(MPa\right) = c1s\theta + c2s + \Delta M_R \tag{4}$$

in which θ is the total stress (kPa) and *c1s* and *c2s* are regression parameters for the saturated state and ΔM_R is the resilient modulus variation from the saturated state to any saturation degree. As mentioned, the databases used include M_R values characterized according to the LC 22-400 standard which allows testing the materials at three water contents. In this study, the regression analysis was performed with the sample characterized at the saturated water content. According to Doucet and Doré (2004), this is necessary to compare all the samples at equivalent matric suction. Therefore, the proposed model firstly estimates the saturated resilient modulus at zero matric suction (Doucet and Doré 2004) and a resilient modulus increase is calculated using a method proposed by Bilodeau (2009).

4.1. Determination of relevant soil properties to include in the estimation model

In order to identify the physical properties statistically associated with *c1s* and *c2s*, correlation matrixes were calculated to obtain guidelines on the choice of the appropriate parameters to include in the model first approximation. Conceptually, the estimation model should include, in addition to the total stress, physical parameters linked with gradation, water content and density. Therefore, in order to consider the water content effect on the M_R, the saturation degree *Sat*, optimal water content W_{opt} and water content *w* were included in the model. For the density, the porosity *n*, dry density *ρd* and maximum dry density *ρdmax* were included in the model. Finally, the effect of gradation is considered through the use of uniformity coefficient *Cu*, fine fraction porosity *nf* and coarse fraction porosity *nc*. The last two, as defined by Côté and Konrad (2003), are obtained with

$$nc = n + (1-n)\%F \tag{5}$$

$$nf = \frac{n}{n + (1-n)\%F} = \frac{n}{nc} \tag{6}$$

in which %F is the fine particles percentage. Table 4 presents the maximum and minimum values of each explanatory variable used to precise the validity of the model.

Table 4. Maximum and minimum values of each parameter

Property	Minimum	Maximum
Cu	3	117
n (%)	13	36
nf (%)	55	93
nc (%)	17.5	43
Sat (%)	13	39
Wopt (%)	4.3	13.1
W (%)	2.5	5.3
ρdmax (kg/m^3)	1579	2442
ρd (kg/m^3)	1686	2280

The regression analysis was performed with two statistic softwares, XLStat and SAS. Using both softwares, estimation models of *c1s* and *c2s* for typical Canadian unbound granular materials used as pavement base and subbase. Good determination coefficients R^2 were found for both estimation models. The values of *c1s* and *c2s* are estimated with

$$c1s = -8.9762 - 0.50796 * \left(\frac{C_u}{n_f}\right) + 0.02717 * S_{at} + 7.7255 * \left(\frac{\rho_{dmax}}{\rho_d}\right) - 0.10661 * w + 0.16825 * w_{opt} \tag{7}$$

$R^2 = 0.74$, R^2 Adj = 0.68

$$c2s = 672.47622 + 14.35277 * n - 6.21307 * n_c + 2.06996 * S_{at} + 0.25323 * \rho_{d\max} - 0.55721 * \rho_d - 27.53323 * w \tag{8}$$

$R^2 = 0.80$, R^2 Adj = 0.73

for which R^2 Adj is the adjusted determination coefficient.

4.2. Calculation of ΔM_R

Using some data used in this study, Bilodeau (2009) proposed a simple method to describe the resilient modulus variation with the changes in saturation degree. This method also describes the resilient modulus water sensitivity according to fine fraction porosity and total stress. From this research, the ΔM_R (MPa) value is described as the product of the saturation degree variation ΔS_R (%) and S. The latter is the slope of the assumed linear relationship between ΔM_R and ΔS_R and is used to obtain the resilient modulus increase from the saturated state in

$$\begin{aligned} \Delta M_R(MPa) &= S \times \Delta S_R \\ \Delta S_R(\%) &= S_R - S_{Rsat} \approx S_R - 100 \end{aligned} \tag{8}$$

for which S is a negative value (MPa/%). Using the actual database, the adapted S model was computed and is expressed as a function of fine fraction porosity and total stress with

$$S\left(\frac{MPa}{\%}\right) = (0.000003\theta + 0.0206)(n_f - 113.636) + 0.31818 \tag{9}$$

in which nf is in percent and θ is in kPa. Figure 1 presents the results calculated with equations 4, 6, 7, 8 and 9 for the data at saturated, initial and drained water contents.

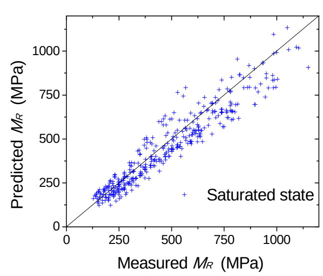
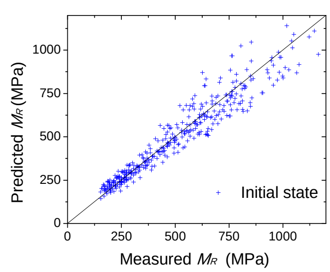
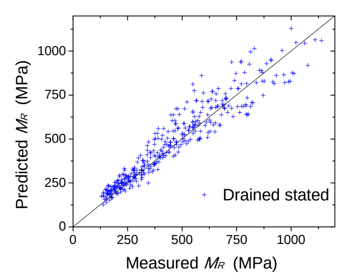
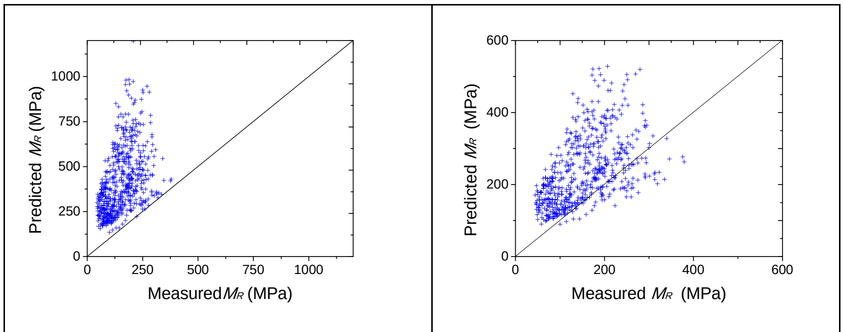
		
Measured MR vs predicted, saturated state. RMSE 15,83%	Measured MR vs predicted, initial state. RMSE 16,14%	Measured MR vs predicted, drained state. RMSE 15,01%

Figure 1. Predicted M_R according to measured M_R

4.3. Model validation

In order to validate the proposed model defined in this study, an independent database was extracted from the data collected throughout the SHRP program (Strategic Highway Research Program). Forty resilient modulus tests on various unbound granular materials from United States and Canada were used. A significant difference was found between predicted and measured resilient modulus for these validation granular materials. This difference is explained by two main differences which are the test methods and the sample water contents. These factors were corrected and a more satisfying trend was found between the predicted and measured resilient modulus as presented in Figure 2.



Measured M_R vs predicted M_R, before the correction. RMSE 74,15%	Measured M_R vs predicted M_R, after the correction. RMSE 42,99%

Figure2. Predicted M_R according to measured M_R, SHRP program.

5. DISCUSSION

The linear model used in this study allows obtaining satisfying saturated resilient modulus estimations for typical Canadian unbound granular materials. It should be pointed out that, in this study, several research efforts were focused on having a simple model that is easy to use with typical geotechnical aggregate characterization tests such as gradation, modified proctor and aggregate density. This research allowed observing the important effect of these granular materials properties on the resilient modulus. In addition, the choice of a linear model was also done for simplicity reasons and non linear models were not retained for this study. The implementation of the ΔM_R value for the considered set of data allows to calculate the resilient modulus at any saturation degree and allows taking into account the effect of fine fraction porosity and total stress to compute the effect of water content on the resilient modulus.

6. CONCLUSION AND FUTURE RESEARCH

The numerous resilient modulus estimation models found in the pavement materials scientific literature shows the various issues encountered to measure this important design parameter. The proposed resilient modulus estimation model gives a first statistically significant approximation for the base and subbase unbound granular materials in comparison to the default values often used in many agencies. The model is valid for large variations of granular materials physical and state properties, but is inadequate for fine-grained soils. An estimation model such as the one presented in this paper has the advantage of considering physical and state properties of aggregate assemblies to estimate the resilient modulus, therefore it is more material adapted. Future researches should focus on revisiting the proposed model to consider non linear models while working with the same general principles. This may increase the predictive capacity of such a model and the estimation confidence level, which is of great interest if the use of a prediction model is to be preferred to a costly direct measurement in a triaxial cell.

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