Asphalt Concrete Pavement Response and Fatigue Performance Modeling Using Advanced Techniques

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ABSTRACT: Modeling the response and performance of asphalt concrete pavements is a critical task for selecting an appropriate pavement design. Current modeling relies on multilayered elastic analysis to extract the pavement response and empirical relationships between these responses and distresses. A more fundamentally appropriate method for this modeling is to couple the response and material models. In this approach, a material model is developed independent of the structure and then coded into a structural model, such as for finite elements. The main advantage of this approach is that the effect of pavement degradation is always explicitly taken into account in the material response. The major limitation of this approach is that it currently requires significant computational time and effort and cannot be used for routine design and analysis. Possible improvement to the current mechanistic-empirical approach can be made by changing the way the pavement response is predicted and changing the model that relates these responses to material performance. In this paper, three well-known pavement response tools – multilayered elastic analysis, multilayered viscoelastic analysis and 3-D finite element modeling – are applied to find the critical responses for a range of pavement structures. Comparisons of each method, along with a discussion of the merits and pitfalls of each, are shown. Then, the pavement responses determined via the two-layered analysis methods are combined with a well-known material performance model, the viscoelastic continuum damage model, to predict the performance of Federal Highway Administration Accelerated Load Facility pavements.

KEY WORDS: asphalt concrete, linear viscoelastic, anisotropy, strength, ductility, viscoelastic continuum damage model

1 INTRODUCTION

The state of practice in pavement analysis involves the use of layered elastic analysis (LEA) to predict primary pavement responses. These predicted responses may then be combined with empirical relationships in a predictive scheme, such as Miner's law, to further deduce pavement performance (Rao Tangella et al. 1990, ARA 2004). Shortcomings of such approaches can be found in both the level of rigor of the empirical relationships and the level of accuracy of the response modeling. LEA has been used for years as the standard for analysis of asphalt concrete pavement systems. Its longevity, in light of the fact that asphalt concrete is known to be a viscoelastic material and that soils are known to be stress state-dependent, is a reflection of the simplicity of the analysis.

Research efforts under NCHRP 9-19, 1-37A, and 1-40A suggest that true linear viscoelastic (LVE) characterization of asphalt concrete material is the future direction of the field. True LVE material characteristics are included in the recently released NCHRP 1-37A Mechanistic-Empirical Pavement Design Guide (MEPDG) using the dynamic modulus, $|E^*|$; however, given the current state of computational power, analysis needs, and/or the willingness of the pavement engineering community to accept the MEPDG, the dynamic modulus has been combined with LEA for the pavement response modeling platform. This mismatch in theory has complicated the existing analysis techniques and has led to considerable confusion regarding the definitions of frequency and time (Dongre et al. 2005, Al Qadi et al. 2008, Underwood and Kim 2009). The inclusion of the viscoelastic nature of asphalt concrete in a framework consistent with that in the MEPDG, however rigorous its treatment may be, does not guarantee improved accuracy. However, it is felt that considering asphalt concrete as viscoelastic is a step towards improved analytical accuracy.

Currently, state of the art pavement analysis includes the use of finite element-based response modeling that may account for many complicated pavement behaviors, such as nonlinear soil response, the viscoelastic nature of asphalt concrete, etc. The leading edge of material modeling is found in advanced models that mathematically consider key material characteristics even beyond linear viscoelasticity, such as microcrack initiation, coalescence and propagation for fatigue and aggregate interlock, viscoplastic flow, and yield surface recovery for rutting. Together, these conceptual approaches may form the basis for the next generation of pavement design and analysis tools. However, modern-day computational limitations and the need for rapid assessment in order to make routine design decisions make the full and consistent implementation of these approaches inefficient.

Such limitations do not necessarily mean that these analysis techniques cannot aid engineers today. The purpose of this paper is to apply and report on the use of various pavement response tools, including LEA, layered viscoelastic analysis (LVEA), and finite element model (FEM) –based analysis, for the prediction of responses for pavement fatigue performance modeling. Through this effort a potential answer to the question as to how best use the dynamic modulus with LEA is presented. To predict fatigue performance, an advanced material model, the viscoelastic continuum damage (VECD) model, is used. Results from the Federal Highway Administration Accelerated Load Facility (FHWA ALF) study are used to verify the basic modeling approach. This approach can be generalized for generic pavement performance predictions using the basic framework of the MEPDG.

2 STUDY PARAMETERS

Because pavement responses are dependent on many variables, it is important that this analytical study covers a range of external conditions. Four different pavement structures, labeled generically as structures 1-4, were selected and are shown schematically in Figure 1. For each structure, four different temperature gradient profiles, predicted from the Enhanced Integrated Climatic Model (EICM) for Raleigh, North Carolina, are used. The chosen temperature gradients correspond to 6:00 AM and 2:00 PM for the months of January and August. Only a single, typical mixture was chosen for this analysis (Kim et al. 2008) because multiple temperature gradients, and thus different regions of material behavior, are dominant. Comparative simulations were performed using a single wheel load with a contact pressure of 760 kPa and a contact load of 40.13 kN. The analysis simulates a moving wheel load and was conducted for a velocity of 26.82 m/s. For each combination, LEA, LVEA and FEM analysis were performed; however, only the results from LEA and LVEA are compared in this paper.

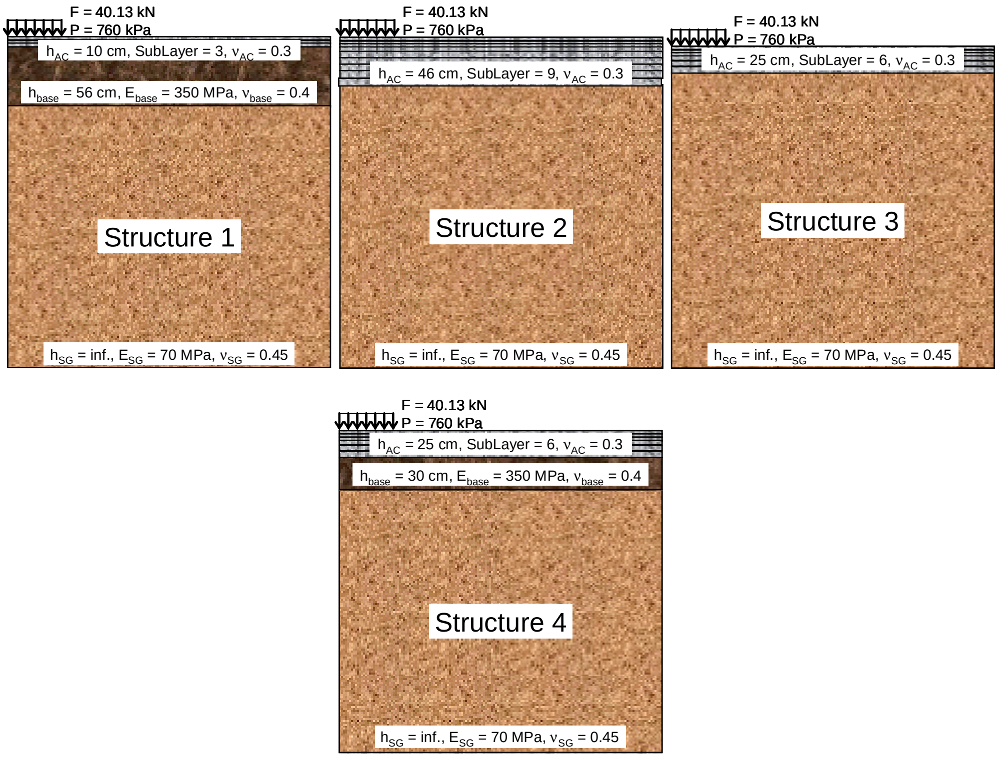

Figure 1: Schematic representation of study pavements.

In the case of viscoelastic analysis, the primary input parameters for the asphalt concrete layers are the Prony coefficients for the axial relaxation modulus, seen in Equation , Poisson's ratio (assumed constant), and the coefficients for the time-temperature shift factor function, seen in Equation . For all analysis the non-asphalt concrete layers are treated as linear elastic.

$$E(t) = E_\infty + \sum_{m=1}^{m} E_m e^{-t/r_m},$$

where E_∞ is the long time elastic modulus; E_m are the Prony coefficients that physically represent the individual spring stiffness values in the Weichert mechanical model; and r_m are the characteristic relaxation times.

$$\log a_T = a_1 T^2 + a_2 T + a_3,$$

where, a_T is the time-temperature shift factor (t-TS) at some temperature T; and a_1, a_2 and a_3 are fitting parameters. The primary input parameters for the unbound layers are the elastic modulus and Poisson's ratio. For both LEA and LVEA, temperature effects are considered by discretizing the asphalt concrete layer into smaller sublayers and applying the temperature distributions predicted from the EICM to each sublayer. In the case of viscoelastic analysis for each of these temperature-constant sublayers, the Prony coefficients are adjusted to match the temperature of that sublayer by applying the shift factor function in Equation . In the case of elastic analysis, the method outlined in a later section of this paper is performed on the discretized structure.

For layered analysis, most simulations were performed using a rectangular wheel load because data suggest that this type of footprint best approximates an actual wheel load (De Beer et al. 2004). In these cases the rectangular footprint is assumed to have a load width equal to 0.6 times the load length. However, because most commercial response tools utilize axisymmetric analysis, and hence a circular wheel load, some limited simulations were performed with a circular wheel load. All analysis tools were developed and implemented by the authors and verified using commercially available response analysis tools, KenPave, Everstress, and WINJulea. This verification is presented elsewhere (Kim et al. 2009) and is not repeated here in the interest of brevity. A naming convention has been developed to simplify the data presentation in this paper. For example, a typical name might be "s1-S-A". The first component of the naming convention is the structure number (s1-s4); the second designates the season (S for summer and W for winter); the third letter denotes morning or evening (A for AM and P for PM); and finally, when circular wheel prints are used, a "c" is at the end of the name.

In the second part of this paper, four of the FHWA ALF pavements are simulated. Each of these pavements is identical to that shown in Figure 1 for Structure 1, but each is constructed using different asphalt concrete materials. The base and subgrade layer moduli are determined via backcalculation from falling weight deflectometer (FWD) data obtained at the actual test sites, and are also given elsewhere (Underwood et al. 2009). For each of these pavements, loading consists of a super-single wheel loaded to 73.8 kN, pressurized at 828 kPa, and moving with a velocity of 4.69 m/s. The temperature gradient is uniform at 19°C across the entire pavement thickness.

3 ANALYSIS USING THE FINITE ELEMENT METHOD

Pavement response and performance analyses using a finite element-based method have some advantages over layered-based analyses, particularly when damage and viscoplasticity are prevalent. This approach is the most accurate because the pavement stress state is complex and is affected not only by the traffic load, but also by other factors: (i) load profile, (ii) boundary conditions, (iii) temperature gradients in the pavement, (iv) nonlinearity of the base and subgrade, (v) stress redistribution due to damage and, when rutting is high due also to rut profile, and (vi) slippage between the pavement and base layers during cracking. The first three of these factors can be taken into account fairly accurately by using layered models. The latter three, though potentially of equal importance, cannot be handled effectively by layered models. A fully consistent approach, which accounts for all six of these factors, consumes a great deal of computational resources. Recognition of this disadvantage is important to understanding and appreciating the role that both FEM and layered analyses play in pavement performance modeling. The alternative to a fully consistent approach is to use FEM analysis to predict the pavement response and then combine this analysis with simplified versions of more comprehensive mechanistic models in a step-wise manner. In this framework, layered analysis results can be utilized in much the same way with considerable computational time savings and with more readily available software. Thus, for the foreseeable future, FEM analysis is seen as a tool to be used to gain insight into specific and detailed problems, whereas its usefulness as a general design and analysis tool seems to be limited.

4 EFFECT OF CIRCULAR VERSUS RECTANGULAR LOAD

In typical layered analysis a circular wheel load is assumed so that axisymmetric analysis can be performed to simplify the necessary calculations. However, recent evidence suggests that a rectangular footprint may better represent actual tire loads. The analysis tools developed for this research can predict the pavement response for either a rectangular or circular wheel load; so, it is of interest to assess the impact of the wheel load footprint. Only the LVEA is used, and only structures 1 and 2 are considered because they represent the extreme asphalt concrete layer thicknesses. The stresses and strains directly under the load center and at the bottom of the pavement are examined because this study focuses on the fatigue phenomenon. The results of this study are summarized in Table 1 where it is seen that for the same external conditions, the circular wheel load increases the peak longitudinal and vertical stresses and strains, but reduces the transverse stresses and strains. In this table and a later table the *% Diff.* is defined as shown in Equation . For the data in Table 1, the rectangular results are taken to be the *reference*. The differences are found to be larger in the thin asphalt section. The practical effect of the strain difference is noticed when the pavement performance is predicted and/or the predictions are calibrated for field data. These differences are likely to be smeared into any lab-to-field calibration process, but because the differences are structurally dependent, and if the performance prediction algorithm is assumed to be appropriate, then an inappropriate load geometry assumption could add to the calibration scatter and reduce the overall reliability of the calibrated performance algorithm.

$$\% \, Diff = \frac{results \text{ - } reference}{reference} \times 100$$

Table 1: Summary of Effect of Wheel Load Type on Pavement Response

Simulation		Longitudinal Strain (με)	Transverse Strain (με)	Vertical Strain (με)	Longitudinal Stress (kPa)	Transverse Stress (kPa)	Vertical Stress (kPa)
s1-S-P	Rect.	149.7	274.3	-232.7	1089.6	1288.7	-309.8
	Circ.	196.8	237.9	-235.4	1345.0	1238.6	-319.3
	% Diff.	31.4	-13.3	1.1	23.4	-3.9	3.1
s1-W-A	Rect.	65.4	86.7	-68.0	2287.5	2567.4	-105.1
	Circ.	78.5	81.3	-71.2	2615.5	2545.7	-109.5
	% Diff.	19.9	-6.2	4.7	14.3	-0.8	4.2
s2-S-P	Rect.	40.1	38.1	-34.7	270.7	248.0	-9.4
	Circ.	42.4	37.3	-35.3	285.4	250.1	-9.1
	% Diff.	5.6	-2.0	1.5	5.4	0.8	-4.0
s2-W-A	Rect.	40.1	38.1	-34.7	270.7	248.0	-9.4
	Circ.	42.4	37.3	-35.3	285.4	250.1	-9.1
	% Diff.	5.6	-2.0	1.5	5.4	0.8	-4.0

5 DETERMINATION OF REPRESENTATIVE ELASTIC MODULUS

The MEPDG developers addressed the issue of inconsistent material and pavement response theories by devising a technique to predict the time of loading for any point in the pavement structure when subjected to a moving wheel load. This method, which is discussed in detail elsewhere (ARA 2004, Underwood and Kim 2009), utilizes the method of equivalent thicknesses in conjunction with Odemark's stress distribution assumption to approximate the time of load for any point along the pavement depth. The pavement structure is discretized into many sublayers, each with a specific temperature, and the process is iterated with the analytical dynamic modulus function until convergence. Figure 2 briefly summarizes the overall procedure. Concerns over this basic procedure exist (NCHRP 1-40A 2006), because in the MEPDG analysis this method suggests a higher modulus value at the surface than in the underlying layers for all conditions, even when the surface is at a high temperature.

Recently, it has been reported that using the dynamic modulus to predict the response of a viscoelastic element subjected to a single stress pulse may result in 20-30% error in the analysis. Instead, a more accurate analysis of the material response can be found by using the so-called *hybrid modulus* defined through rigorous LVE analysis and shown mathematically in Equation .

$$E_H(f) = \frac{|E^*|(f) + E\left(t = \frac{1}{2f}\right)}{2},$$

where, $E_H(f)$ is the hybrid modulus at a particular frequency; $|E^*|(f)$ is the dynamic modulus at the same given frequency; and E is the relaxation modulus at a time equal to the inverse of twice the given frequency.

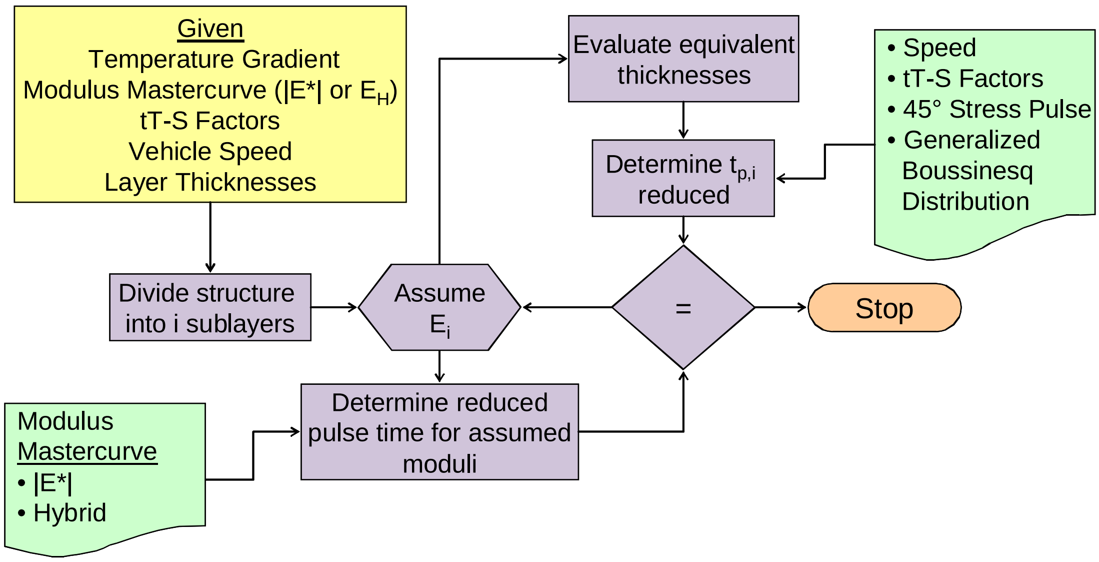

Figure 2: Schematic representation of representative elastic modulus determination procedure.

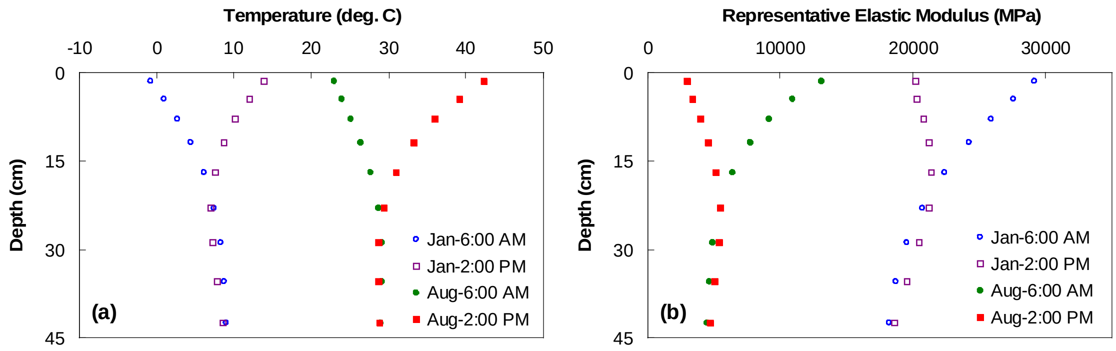

Figure 3: (a) EICM-predicted temperature gradients and (b) representative elastic modulus gradients for structure 2.

For the hybrid modulus the predicted response errors are more acceptable at about 1-5% (Underwood and Kim 2009). This finding has direct implications for the analysis at hand because the MEPDG technique treats a loading cycle as a single passing load pulse. For the analysis conducted in this paper, the situation noted by NCHRP 1-40A researchers for MEPDG analysis arose only when a positive temperature gradient (i.e., cooler at the surface than at the bottom) existed. When a negative temperature gradient existed, the basic trend was an overall lower modulus at the surface, which increased slightly with depth until a point at which the modulus started to decrease with depth. This transition between increasing and decreasing moduli is a result of the counteracting effects of temperature and loading time. The negative temperature gradient alone would suggest that the modulus would increase consistently with depth. Conversely, loading time monotonically decreases with depth and suggests that the modulus should consistently decrease with depth. Figure 3 presents a sample of typically observed behaviors in terms of temperature and modulus gradients.

Table 2: Summary of the Effect of Representative Modulus Determination Method on Two Critical Response Parameters

Simulation		Transverse Strain (% Diff.)	Vertical Stress (% Diff.)	Simulation		Transverse Strain (% Diff.)	Vertical Stress (% Diff.)
s1-S-A	$\|E^*\|$	-12.0	-10.6	s2-W-A-c	$\|E^*\|$	-5.2	-6.9
	E_H	4.3	2.4		E_H	6.8	3.1
s1-S-P	$\|E^*\|$	-13.6	-14.1	s2-W-P	$\|E^*\|$	-9.0	-13.1
	E_H	2.0	-0.8		E_H	3.9	-2.0
s1-S-P-c	$\|E^*\|$	-11.9	-12.4	s3-S-A	$\|E^*\|$	-17.3	-15.5
	E_H	2.5	1.4		E_H	6.2	2.1
s1-W-A	$\|E^*\|$	-4.8	-3.2	s3-S-P	$\|E^*\|$	-21.2	-20.8
	E_H	0.6	0.6		E_H	3.4	-1.3
s1-W-A-c	$\|E^*\|$	-4.8	-2.8	s3-W-A	$\|E^*\|$	-3.4	-3.0
	E_H	0.6	1.1		E_H	6.2	3.6
s1-W-P	$\|E^*\|$	-8.1	-8.0	s3-W-P	$\|E^*\|$	-11.4	-13.0
	E_H	-0.2	-1.2		E_H	0.0	-4.0
s2-S-A	$\|E^*\|$	-16.0	-18.2	s4-S-A	$\|E^*\|$	-17.3	-14.2
	E_H	11.3	3.3		E_H	5.0	4.0
s2-S-P	$\|E^*\|$	-20.4	-22.8	s4-S-P	$\|E^*\|$	-20.9	-19.4
	E_H	6.9	-1.2		E_H	2.0	0.6
s2-S-P-c	$\|E^*\|$	-20.4	-24.4	s4-W-A	$\|E^*\|$	-6.9	-6.3
	E_H	6.9	-1.7		E_H	2.2	0.2
s2-W-A	$\|E^*\|$	-5.4	-7.5	s4-W-P	$\|E^*\|$	-11.3	-12.2
	E_H	6.5	2.0		E_H	-0.1	-3.1

The findings for two key responses, transverse strain and vertical compressive stress, at the bottom of the asphalt concrete layer for the different simulation conditions are given in Table 2. The results are shown as the *% Diff* from Equation with the LVEA simulation results being the *reference* and the LEA simulation results being the *results*. The MEPDG procedure for determining the representative modulus value is based upon the vertical stress distribution within the pavement system. As a result, it is expected that this response parameter would be best matched by the elastic analysis. Indeed, the data in Table 2 show that for the hybrid modulus the difference is relatively small, at most 4% for the test conditions. However, as expected from the results given elsewhere (Underwood and Kim 2009) the dynamic modulus-based analysis shows much higher errors, up to approximately 25%. Even though the hybrid modulus-based analysis better matches the vertical stresses at the bottom of the pavement system, it does not predict the lateral and transverse stresses as well as the dynamic modulus-based approach. The reason for this behavior is believed to be related to the fact that, in general, transverse and longitudinal stress pulses are shorter in duration than vertical stress pulses. Because the dynamic modulus is greater than the hybrid modulus for a fixed temperature and frequency, the net result is a more accurate prediction of the effective transverse and longitudinal moduli and thus lowers errors in these stresses. The net effect of these combined errors are, interestingly enough, a lower overall error in the longitudinal strains for the dynamic modulus-based analysis (average of -1% versus 15.4%), but a higher overall error in the transverse strains for the dynamic modulus-based analysis (-12% versus 3.8%). Although not shown here, it is also noted that errors in vertical strain at the bottom of the subgrade are similar in magnitude for both approaches (-7.5% for $|E^*|$ versus 10% for E_H). It should be strongly noted that these results are only for the strains at the bottom of the asphalt concrete layer. Although not assessed for this study, strain comparisons at other locations within the pavement structure may not yield similar error values; thus, caution must be taken in generalizing these findings in such a way.

6 SIMULATION OF FHWA ALF PAVEMENTS

As noted previously, the most accurate method of modeling pavement performance is to use a fully consistent approach wherein the state and evolution of key factors are taken into account at each calculation step. However, such analysis is practically impossible for routine design and analysis decisions and, instead, a simplified model scheme that uses the output from the response model simulations and a new derivation of the VECD model has been applied.

Continuum damage theories ignore specific microscale behaviors and attempt to characterize a material using the net effect of these microstructural changes on observable properties. These theories can accomplish this task efficiently by tracking the instantaneous secant modulus, *C*. Because the continuum damage model used in this study takes advantage of the elastic-viscoelastic correspondence principle, the instantaneous secant modulus used here is the one in stress-pseudo strain space. Damage is oftentimes more difficult to quantify and generally relies on the combination of macroscale measurements and rigorous theoretical models. Space limitations do not allow for a full discussion of these issues here, but a complete discussion of the model, including material level verification, is given elsewhere (Kim et al. 2008, Underwood et al. 2009). For the purposes of this paper, it is only important to understand that the VECD model has been formulated in such a way that the response model results may be used to predict the amount of damage that will accumulate over a specified number of repetitions. The VECD model can be characterized using either constant rate tension or cyclic fatigue tests until failure in direct tension to characterize the damage behavior, and conducting temperature and frequency sweep experiments at relatively low magnitudes to characterize the LVE behavior.

For the pavement performance simulation, the pavement response to a single wheel pass is first determined using either LEA or LVEA. The VECD model is then used to predict the response of a mixture sample that is subjected repeatedly to this strain kernel. When the simulated *C* value degrades from an initial value of 1 to 0.25 then the simulation is stopped, and the cycle number is recorded as the failure cycle. Comparisons are then drawn between this failure cycle and the number of cycles to failure of a full-scale pavement, in this case the FHWA ALF pavements.

The results of the test simulations are shown for the available ALF pavements in the plots in Figure 4. Based on the results of Kutay et al. (2008), failure in the ALF pavements is defined as the cycle when 20% of the lane has cracked. Note that the Terpolymer mixture is not included in establishing the best fit lines in these graphs. It has been noted by other researchers that certain problems relating to the distribution of hydrated lime throughout the Terpolymer test lane, among other factors, may have affected the ALF test results (Kutay et al. 2008). Figure 4 shows that the predicted and measured fatigue lives are consistent for each response prediction, although the exact relationship between the two varies with methodology. The magnitude error could be caused by laboratory-to-field differences (all experiments were conducted on laboratory-mixed and gyratory-compacted samples), the use of only the initial material response, or model errors, and should be the subject of ongoing study. Regardless of the cause of the error, it is clear from Figure 4 that the measured and predicted fatigue lives are consistent and that the modeling approach could yield accurate predictions of the pavement fatigue life with an appropriately calibrated transfer function. One may argue that because the predictability, after applying an appropriate transfer function, does not seem to be affected by the response modeling technique, LVEA analysis is not necessarily worth the effort. With this argument, any differences that may exist would be taken into account in the calibration process. However, as with the effect of circular versus rectangular wheel load, the differences shown in Figure 4 are structurally dependent and may be exacerbated (or improved) under certain conditions. The net result of this effect would be an increase in calibration scatter and a reduction in the overall reliability of the performance prediction algorithm.

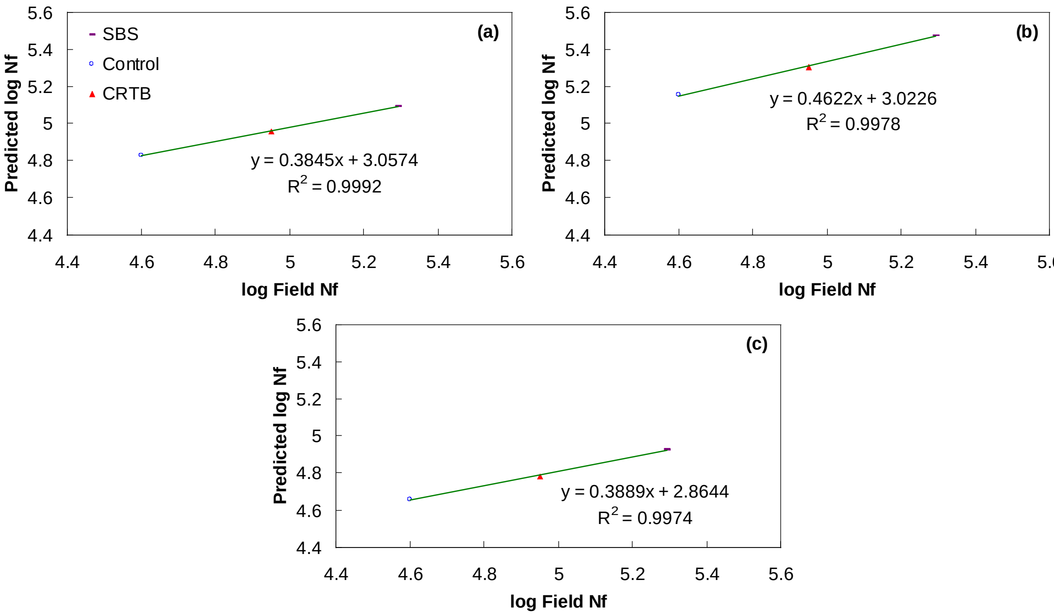

Figure 4: Comparison of measured and predicted cycles to failure in ALF experiment with best fit function for: (a) LVEA, (b) LEA with |E*| analysis, and (c) LEA with E_H analysis.

7 SUMMARY AND CONCLUSIONS

A numerical study of the behavior of asphalt concrete pavements subjected to moving wheel loads has been performed. This study focuses on the benefits and shortcomings of three pavement response tools: LEA, LVEA, and FEM analysis. The findings from this study suggest that the use of the FEM for complete pavement response and performance prediction is still too computationally intensive for routine pavement design and analysis. As a result, it is believed that simpler response prediction tools, such as LEA or LVEA, will be important to mechanistic pavement analysis for the foreseeable future. Although LVEA is beneficial because of its more accurate treatment of asphalt concrete layers, it is not as readily accessible to engineers as LEA tools. The results of this study show that for pavement responses at the bottom of the asphalt concrete layers, LEA can be performed such that it yields reasonably accurate values as compared to LVEA. However, to do so, care must be taken in selecting the appropriate modulus value to use. Finally, a pavement performance algorithm is presented that uses the simple pavement response properties in conjunction with the VECD material model. This analysis technique is verified using results from the FHWA ALF experiment. Although not shown in this paper, the algorithm can be generalized and used to predict generic pavement performance using the same principles followed in the MEPDG.

8 ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support provided by the Federal Highway Administration.

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