A Practical Method for Converting Resilient Modulus to Dynamic Modulus of Asphalt Mixtures

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ABSTRACT: With the impending implementation of the AASHTO Mechanistic-Empirical Pavement Design Guide (MEPDG), dynamic modulus ($|E^*|$) of hot-mix asphalt (HMA) has been selected as the basic material property input for asphalt pavement analysis and design. The resilient modulus (M_R) of HMA used in the previous AASHTO pavement design guide (1993 and before) will gradually become outdated and useless. Therefore, there is a need to convert the already existing M_R databases to $|E^*|$ databases so that these M_R data can still be used for future analysis with the new mechanistic-empirical method. This paper presents a practical method for converting M_R to $|E^*|$ of HMA mixtures. Based on the analysis and comparison of master curves of M_R and $|E^*|$, the master curve of M_R can be constructed by fitting a quadratic function to the three test data points obtained at three test temperatures used in resilient modulus test. Then the master curve of $|E^*|$ is obtained by slightly shifting upward the master curve of M_R . The comparison of the predicted and measured $|E^*|$ values from the HMA mixtures used in the U.S. shows that this method was reasonably effective in predicting $|E^*|$ values from existing M_R measurements.

KEY WORDS: Dynamic modulus, resilient modulus, conversion, asphalt mixtures.

1 INTRODUCTION

With the transition from the 1993 Pavement Design Guide to the new Mechanistic-Empirical Pavement Design Guide (MEPDG) of the American Association of State Highway and Transportation Officials (AASHTO), the widely used hot-mix asphalt (HMA) material property in previous AASHTO design guides (1993 and before), resilient modulus M_R , has been replaced with the newly introduced material property, dynamic modulus ($|E^*|$) (AASHTO 1993, NCHRP 2004). This significant change from M_R to $|E^*|$ will make the already existing M_R databases in many highway agencies outdated and useless in the MEPDG. Nowadays, research efforts in many highway agencies are directed to develop their own databases of $|E^*|$ from laboratory experiments.

However, if $|E^*|$ values can be predicted from the existing M_R databases based on the relationship between the materials properties of HMA, the $|E^*|$ database can be developed without (or with much reduced amount of) laboratory experiments. Thus, significant research budget and effort can be saved. To achieve this goal, many pavement researchers have

attempted to develop a correlation between M_R and |E*|. Flintsch et al. (2005) and Loulizi et al. (2006) compared M_R and |E^{*|} from two typical HMA mixtures used in Virginia, USA and found a strong correlation between M_R and $|E^*|$ at the frequency of 5 Hz. Ping and Xiao (2007, 2008) evaluated the M_R and $|E^*|$ results from the asphalt mixtures used in Florida, USA. They found that $|E^*|$ increases with an increase in M_R at a specific loading frequency and the M_R values are comparable with the $|E^*|$ values at the loading frequency of 4 Hz. Lacroix et al. (2007) proposed an analytical method for predicting M_R from $|E^*|$ based on the linear viscoelastic theory. They found that the measured and predicted M_R values from four mixtures made with different gradations and binder types are in close agreement. Later, Lacroix et al. (2008) developed an M_R database from the existing Witczak $|E^*|$ database using their proposed method. With an artificial neural network (ANN), they then established the relationship between these two databases and backcalculated $|E^*|$ values from M_R database. They verified the ANN model with an independent |E*| database for the mixtures of North Carolina, USA. They found that the backcalculated |E*| are reasonably comparable to the measured values. However, both the empirical correlation method and the ANN method can only predict |E*| through the point-to-point conversion and did not take advantage of other useful material property information (such as master curve of the moduli), which may have a negative effect on the conversion accuracy and thus limit their practical application.

2 BACKGROUND

In mechanistic-empirical or mechanistic pavement design methods, stiffness or modulus of HMA is a basic material property input that is used to calculate the stress and strain responses of asphalt pavements under traffic loads. Since the introduction of asphalt stiffness by Van der Poel (1954) to describe the viscoelastic properties of asphalt mixtures, various stiffness concepts and testing methods have been proposed to characterize the HMA stiffness properties. Some of these stiffness properties are creep compliance, relaxation modulus, complex modulus (or dynamic modulus and phase angle), and resilient modulus (Kim and Lee 1996, Hu et al. 2008). Different loading modes (compression or tension) and stress states (uniaxial or biaxial) are employed in the testing methods to perform the tests (Kim and Lee 1996, Hu et al. 2008).

Based on the theory of linear viscoelasticity, three essential material functions are used to characterize the properties of a viscoelastic material. They are creep compliance D, relaxation modulus E, and complex modulus E* (Ferry 1980, Tschoegl 1989). Since these three material functions contain the same information that governs the stress strain behavior of a viscoelastic material, there exist inherent interrelationships among them (Kim and Lee 1996, Ferry 1980, Tschoegl 1989, Park and Schapery 1999), which provide the basis for the interconversion between these three basic material functions.

Many researchers in the asphalt industry have investigated the interconversion between the linear viscoelastic material functions and evaluated their applications to HMA mixtures. Detailed information can be found in Park and Schapery (1999), Schapery and Park (1999), Kim and Lee (1996), Daniel and Kim (1998), Park and Kim (1999), Dongré et al. (2006), and Katicha et al. (2008).

3 COMPARISON BETWEEN M_R AND $|E^*|$ FOR HMA MIXTURES

In the asphalt industry, the commonly used method for dynamic modulus test is to apply a uniaxial cyclic compressive loading to a specimen. This cyclic load is not a true sinusoidal

load as required for the regular dynamic modulus test. Instead, the compressive cyclic load can be considered as a combination of a true sinusoidal load (alternating compressive and tensile load) and a constant compressive load as in a creep test. Therefore, the resulting compressive strain is the sum of the sinusoidally changing strain response and the creep strain caused by the constant compressive load. To eliminate the influence of the creep strain, a nonlinear regression is usually required to fit the following function to the measured data to obtain the strain response amplitude ε_0 :

$$\varepsilon(t) = a_0 + b_0 t + \varepsilon_0 \sin(\omega t - \phi) \tag{1}$$

where a_0 , b_0 = regression constants.

In the HMA resilient modulus test, a haversine compressive load is usually applied to a specimen followed by a rest period. This load can be decomposed into a combination of a cycle of true sinusoidal load and a constant load. It should be noted that the recoverable strain occurring during the unloading and the rest period is used to calculate the M_R value for the resilient modulus test. Due to the existence of the rest period, even if stress level and loading frequency are the same for both dynamic modulus and resilient modulus tests, the recovered strain in the resilient modulus test is somewhat larger than the strain amplitude in dynamic modulus test (Figure 1). This means that the measured M_R value will be lower than the $|E^*|$ value at the same loading frequency. This phenomenon has been verified by many laboratory test results (Loulizi et al. 2006, Flintsch et al. 2005, Ping and Xiao 2007, 2008, Hu et al. 2008).



Figure 1: Comparison of strain responses in M_R and $|E^*|$ tests

According to the studies by Pellinen and Witczak (2002) and Pellinen (2001), master curve of dynamic modulus of HMA can be constructed by fitting the following sigmoidal function to the measured data:

$$\log|E^*| = \delta + \frac{\alpha}{1 + e^{\beta + \gamma \log f_r}}$$
(2)

where $\log |E^*| = \log$ of dynamic modulus; $\delta = \min \mod \operatorname{modulus}$ value; $f_r = \operatorname{reduced}$

frequency; α = span of modulus value; and β , γ = regression constants indicative of the shape of the sigmoidal function.

The reduced frequency, f_r , at a test temperature, T, can be obtained from the following equation (Pellinen and Witczak 2002):

$$f_r = a_T f$$
 or $\log f_r = \log a_T + \log f$ (3)

where, $a_{\rm T}$ = shift factor; and f = loading frequency.

The shift factor defines the required horizontal shift from a test temperature, T, to an arbitrarily selected reference temperature of the master curve. The shift factor is a function of temperature, which can be expressed as (Hu et al. 2008):

$$\log a_T = a_1 T^2 + b_1 T + c_1 \tag{4}$$

where, a_1 , b_1 , c_1 = regression coefficients.

Hu et al. (2008) compared the dynamic modulus and resilient modulus values from four mixtures used in Texas, USA (mixture labels: 1/2A, 1A, FC1, and Trap1) at various test temperatures and loading frequencies. The mixtures were produced with three asphalt binders (PG 64-22, PG 70-22, and PG 76-22) and three aggregates (crushed river gravel, limestone, and basalt). Both dynamic and resilient modulus results from Hu et al. (2008) were used in this study to fit Equation (2) to construct the master curves for dynamic and resilient moduli using a nonlinear least square regression method. The nonlinear regression was performed using the Microsoft Excel Solver add-in function. The seven regression parameters (δ , α , β , γ , a_1 , b_1 , and c_1) can be simultaneously determined during the nonlinear regression process. Figure 2 shows the comparison of the master curves for dynamic and resilient moduli. Figure 3 shows the shift factors at different test temperatures for the four mixtures.

From Figures 2 and 3, the following observations were obtained:

- (a) The measured data of both dynamic modulus and resilient modulus of HMA mixtures could fit the sigmoidal function (Equation 2) very well within the range of the measured data. The coefficients of determination, R^2 , for the nonlinear regression for both moduli of four mixtures were high up to 0.999, indicating that they were very good regressions. Therefore, not only the master curve of dynamic modulus, but also the master curve of resilient modulus of HMA mixtures can be constructed by fitting Equation (2) to the test data. The R^2 of the best fit for the shift factors of all four mixtures is nearly equal to one, which means that the shift factors can be represented with Equation (4).
- (b) As explained in the previous section, M_R value is always lower than $|E^*|$ for the same HMA mixture at the same test temperature and loading frequency. It is observed from Figure 2 that the master curves for M_R and $|E^*|$ were not parallel to each other, which indicates that the difference between $|E^*|$ and M_R varies with temperature and loading frequency. The difference was larger at high loading frequencies (or low temperatures) than at low loading frequencies (or high temperatures). The average ratio of the measured $|E^*|$ to M_R at the same temperature and loading frequency was about 1.3 for the four mixtures. Therefore, the curve of 1.3 times the fitted M_R value vs. frequency was also presented and compared to the measured $|E^*|$ values in Figure 2. It can be seen that the value of 1.3*(fitted M_R) was generally in close agreement with the measured $|E^*|$ values.
- (c) The shift factor curves of dynamic modulus and resilient modulus were distributed closely to each another except for the one of dynamic modulus master curve for Trap1 mixture. This means that the shift factors can be predicted from the average of the shift factor curves. Since the resilient modulus test is usually performed at three temperatures, 5°C,

25°C, and 40°C, the logarithmic values for the shift factors at these three temperatures were determined to be 3, 0 (as reference temperature), and -2, respectively, based on the average of the predicted values from the shift factor curves.



Figure 2: Master curves of $|E^*|$ and M_R for Texas HMA mixtures



Figure 3: Shift factors of |E*| and M_R for Texas HMA mixtures

However, even if the three sets of $|E^*|$ values and the reduced frequencies can be predicted from the measured M_R values at three test temperatures, the master curve for $|E^*|$ cannot be constructed with the three data points because at least four data points are required for the nonlinear regression to obtain the four regression parameters (δ , α , β , and γ). Further investigation into the M_R and $|E^*|$ data points of the four Texas HMA mixtures shows that both can be fitted to a quadratic equation:

$$\log |E^*| \ (\operatorname{or} \log M_R) = a_2 (\log f_r)^2 + b_2 (\log f_r) + c_2 \tag{5}$$

where, a_2 , b_2 , c_2 = regression coefficients.

As an example, Figure 4 shows the shifted data points of resilient modulus results for 1/2A mixture and the curve of the fitted quadratic equation. It can be seen that a quadratic function can be well fitted to the shifted data of M_R results. The R^2 is greater than 0.99, indicating it is an excellent regression.



Figure4: Quadratic fit of dynamic modulus master curve of Texas 1/2 mixture

4 PRACTICAL METHOD FOR CONVERTING M_R TO |E*|

Based on the previous analysis, the proposed practical procedures to convert M_R of HMA mixtures to $|E^*|$ can be summarized as follows:

- (1) Determine the shift factors at the three test temperatures of resilient modulus test based on the information about asphalt binder and mixture. Dongré et al. (2006) and Lacroix et al. (2008) verified that the shift factors for the master curve of HMA mixtures are virtually the same as those for the asphalt binders used to produce the HMA mixture. Therefore, reliable shift factor values for HMA mixtures can be obtained from those of asphalt binder. If this information is missing, shift factors at 5°C, 25°C, and 40°C are about 3, 0, and −2, respectively, if the temperature of 25°C is used as the reference temperature for the construction of master curve.
- (2) Construct the master curve of resilient modulus by fitting the quadratic function Equation(5) to the three data points obtained from three different test temperatures used in resilient

modulus test. Because the master curve is developed within the temperature range of 5° C and 40° C, caution should be exercised if the predicted dynamic modulus is obtained by extrapolating the fitted curve beyond the test temperature range.

(3) Shift the fitted master curve of resilient modulus upward to certain extent to obtain the master curve of dynamic modulus. This means that the dynamic modulus value can be acquired by multiplying the estimated resilient modulus by a factor:

$$|E^*| = \beta_0 M_{R, \text{ predicted}} \tag{6}$$

where, β_0 = multiplication factor; and $M_{R, \text{ predicted}}$ = predicted M_R value from the constructed master curve of resilient modulus.

The multiplication factor, β_0 , defines the ratio of measured $|E^*|$ to M_R at the same temperature and loading frequency. From the comparison between measured M_R and $|E^*|$ values in Figure 2, the average ratio of $|E^*|$ to M_R for the Texas mixtures is about 1.30. However, it should be noted that $|E^*|$ and M_R of asphalt mixtures are influenced by many factors, such as the properties of asphalt binder and aggregate, asphalt content, air voids, aggregate gradation, etc., and their effects on $|E^*|$ and M_R may be different. Therefore, the factor β_0 may vary for different HMA mixtures.

5 VALIDATION OF THE PROPOSED METHOD

Ping and Xiao (2007, 2008) conducted a systematic study to evaluate and compare the dynamic modulus and resilient modulus for the HMA mixtures used in Florida, USA. They selected 20 Superpave asphalt mixtures for the evaluation. The 20 mixtures used one type of asphalt binder, PG 67-22, and different types of aggregate: 14 Georgia granite materials, one Nova Scotia granite, one North Florida limestone, two Central Florida limestone, one South Florida oolite, and one Alabama limestone. The nominal maximum aggregate sizes for the mixtures were 19.0 mm, 12.5 mm, and 9.5 mm, respectively. The asphalt content for the mixtures varied from 4.5 to 8.2. The air void contents of the specimen for both dynamic and resilient modulus tests were controlled to be within $4 \pm 0.5\%$. Both tests were performed at three test temperatures (5 °C, 25 °C, and 40°C). The loading for the resilient modulus test was a 0.1 second haversine load pulse followed by a rest period of 0.9 s. Other information about the mixtures and the testing can be found in Ping and Xiao (2007, 2008).

Since no information is available for the shift factors of resilient modulus, the logarithmic values of shift factors (log a_T) were estimated from Figure 3 to be 3, 0, and -2 at 5 °C, 25 °C, and 40°C, respectively. The multiplication factor (β_0) value of 1.3 was used for all the mixtures. Then the dynamic moduli and their master curves were predicted following the procedures proposed in this study.

Figure 5 shows the comparison between the master curves of the predicted and measured $|E^*|$ from two of the 20 mixtures (S-12 and S-14 mixtures). Fairly good agreement was observed between the predicted and measured $|E^*|$ values for both mixtures. Within the range of the loading frequencies, the master curves for both resilient modulus and dynamic modulus followed the trend of a quadratic function in the form of Equation (5).

Figure 6 shows the overall comparison of the predicted and measured $|E^*|$ values from all 20 Florida mixtures. Even though the shift factors were estimated from Figure 3 and the multiplication factor (β_0) of 1.3 were used in the prediction for all the mixtures, a reasonably close agreement could be observed between the predicted and measured $|E^*|$, which verified the applicability of the proposed method.



(b) SM-14 Mixture Figure 5: Comparison between $|E^{\ast}|$ and M_{R} for Florida mixtures



Figure6: Comparison between predicted and measured |E*| for Florida mixtures

6 SUMMARY AND CONCLUSIONS

A practical method was proposed to convert M_R to $|E^*|$ for HMA mixtures with the construction of mater curve. The method involves three steps: (1) determine the shift factors of M_R at different test temperatures to convert the actual loading frequency to the reduced frequency; (2) construct the master curve of M_R by fitting a quadratic equation to the three test data points; (3) shift vertically upward the master curve of M_R to certain extent (i.e. $\log \beta_0$) to obtain the master curve of $|E^*|$ and then the $|E^*|$ values can be estimated at different loading frequencies. Based on this study, the following conclusions and recommendations can be summarized:

- The master curve of both dynamic modulus and resilient modulus of HMA mixtures can be expressed using the sigmoidal-shaped function in the form of Equation (2).
- The master curve for resilient modulus and dynamic modulus can also be approximated with a quadratic function in the form of Equation (5).
- Unlike the sigmoidal-shaped function, the quadratic function cannot reflect the S-shape of master curve. Therefore, the proposed converting method may not be valid for extreme temperatures or loading frequencies when HMA mixtures have already reached their minimum or maximum modulus values.
- Caution should be exercised if the |E*| values are predicted by extrapolating the fitted master curve of |E*| beyond the ranges of test temperatures or loading frequencies.
- The values of the shift factors and the multiplication factor (β_0) were influenced by many factors associated with the composition and properties of HMA mixtures and should be determined desirably based on relevant mixture information. In this study, the multiplication factor is estimated to be $\beta_0 = 1.3$ for HMA mixtures used in Texas and Florida, USA.
- This is a preliminary study for converting M_R to $|E^*|$ for HMA mixtures with the assistance of master curve. The proposed converting method should be validated using more HMA mixtures before it can be put into application.

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