Survival Analysis of Rutting for Flexible Pavements Based on LTPP Sections

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ABSTRACT: Flexible pavement performance models are a common component of design, analysis and management procedures used by highway agencies. Most models are structured to predict performance as a function of a set of covariates that are expected to have an effect. Other models are intended to predict time to failure (given a specific failure criterion, e.g. 0.50 in total surface rut depth). This later set of models is of special interest in pavement management and preservation programs. The variable of interest in the later models is a variable event; given a set of homogeneous pavement sections, failure will be reached at different times in different locations. Additionally, time to failure is not constantly monitored but can be obtained from pavement surveys that are carried out on a periodical basis. This means that there is data censoring and truncation (failure events that are not observed) that has to be considered in modeling time to failure so that the model estimates are not biased.

A survival analysis of a subset of the data collected on flexible pavement sections from the Long Term Pavement Program database will be performed. The authors propose a framework to estimate pavement survival models, by means of a parametric and a semi-parametric approach, as a function of exogenous variables. The analysis indicated that traffic, asphalt layer thickness, the asphalt mix properties (asphalt and air void content), and an the number of days a pavement structure is below 32°F have the most significant effect on the survival probability and failure rate of a pavement section due to rutting. Additionally, a comparison between the parametric and semi-parametric approached is presented.

KEY WORDS: Survival models, failure rate, hazard ratio, LTPP, rutting.

1 INTRODUCTION

Pavement performance models are commonly involved in the design of new pavement structures and in the analysis and management of existing ones. Performance models currently used can be classified in two distinct categories: purely empirical models and mechanistic-empirical models. Purely empirical models, such as the one included in the 1993 AASHTO Pavement Design Guide (AASHTO, 1993), estimate pavement performance as a function of indicator variables that have been identified as highly correlated with the different distress types such as roughness, cracking, rutting, and patching. On the other hand, mechanistic-empirical models use mechanistic principles to transform loading and environmental variables into expected stresses, strains, and deflection in the pavement structure (mechanistic component) which, in turn, are related to different distress levels by means of statistical models (empirical component). This is the case of the Mechanistic Empirical Pavement

Design Guide or MEPDG (ARA, 2004) developed under the National Cooperative Highway Research Program (NCHRP 1-37A and 1-40D).

Most of the performance models currently in use are also used to quantify the amount of a given type of distress after a given period of time (usually design period). However, we might also be interested in estimating time, or number of applied loads, until a given pavement structure fails or, more specifically, the time until pavement conditions fall below an acceptable level. This type of model can be very useful not only in design, but also in designing preservation, maintenance, and rehabilitation strategies, as well as in budget and resource allocation. However, in estimating the later type of models special care needs to be taken so that the information available can be used in the most efficient way. These types of models are known as survival models, and the considerations involved in estimation fundamentally differ from those used in regular regression models. The main difference lies in that the variable that is being modeled is time (or some indicator of time), as opposed to a given amount of distress.

Although in general, the use of survival models has not been wide-spread in pavement applications, several performance models based on survival analysis have been successfully employed in the past. Winfrey (1969) reported the use of empirical survival models in pavement engineering since the early 1930s. More recently, The World Bank used survival models for estimating cracking initiation or “number of years to the initiation of narrow (or wide) cracks since last surfacing or resurfacing”, and similarly raveling initiation, and potholing initiation included as part of the HDM-III Models (Watanatada et al., 1987). In the last decade more applications of survival analysis have surfaced in pavement engineering. Prozzi et al. (2000) demonstrated that a survival model that follows the same structural form as the one developed using the American Association of State Highway Officials (AASHO) Road Test match the observed pavement data better than the estimates provided by the original AASHO Equation. Wang et al. (2005) also showed the advantage of using a survival model in identifying reasons for premature fatigue cracking, as well as predicting average pavement behavior of flexible pavements by using Long Term Pavement Performance (LTPP) data.

2 SURVIVAL ANALYSIS

2.1 General

“Survival analysis is a class of statistical methods for studying the occurrence and timing of events” (Allison, 1995). Pavement failure is one example of a timed event. Even in the case of pavement sections constructed with similar structures and materials, the time to failure of the pavement sections is not homogeneous. However, as mentioned previously, there are several aspects that need to be considered when analyzing this type of data. As opposed to regular longitudinal data, we are not only interested in knowing whether a pavement structure has failed or not based on some predefined criteria but, more importantly, we need to know when the failure did occur. In other words, we are interested in knowing the exact moment when the pavement failure occurred, e.g. when the surface reached 0.75 rut depth.

The main difficulty involved in analyzing survival data, as compared to conventional regression methods, is related to the censoring of failure observations. Censoring can occur as a result of several reasons. Left censoring occurs when a pavement section has failed before the pavement monitoring or surveying begins. As an example, left censoring cases can be observed on the pavements sections from the General Pavement Studies (GPS) LTPP sections that were constructed several years before they were entered into the LTPP database. It is common to observe that some of these sections have already failed when the first LTPP survey was conducted.

The second type of censoring that can be commonly observed in pavement monitoring databases are pavement sections that have not yet failed when the surveying experiment has finalized; this is known as right censoring. This occurs when no failure was observed and, therefore, all that is known is that the given pavement structure has a time to failure that is greater than the monitoring period.

Finally, the third type of censoring occurs when pavement monitoring or surveying is performed at given time intervals. e.g. every second year. We know that if a pavement section had not failed when the previous survey was conducted, but has been identified as failed on the current survey, that the pavement failure occurred sometime between the times that the surveys were conducted. However, the exact time of failure is not well known: this is commonly referred to as interval censoring.

If the observations that are left or right censored were to be dropped from the analysis, so that only pavement failures that occurred during the given observation period are considered, then the model will be significantly biased because of the data truncation. For example, if the right censored observations were to be excluded, the model estimates would predict lower pavement life expectancy. On the other hand, if the right and left censored observations are included, but are not considered appropriately, the model will suffer from censoring bias.

2.2 Time to Pavement Failure

Under the assumption that the underlying random variable *T* (time to failure of pavement structure) is continuous, the survival function or probability that a pavement structure survives past time *t* can be expressed as $S(t) = 1 - F(t)$ where $F(t)$ is the probability that a pavement structure fails before time *T* = *t*. Then, the density function of *T* can be denoted by $f(t) = dF(t)/dt$.

Subsequently, for $h > 0$, $P(t \leq T \leq t + h | T \geq t)$ is the probability that the pavement structure fails in the interval $[t, t + h)$ given that the pavement structure has survived until time *t* (Wooldridge, 2002). The hazard function for *T* can then be defined as,

$$\lambda(t) = \lim_{h \to 0} P(t \leq T \leq t + h | T \geq t)/h \qquad (1)$$

The hazard rate can be interpreted as the instantaneous rate of pavement failure per unit of time. From (1), for small h, we have that $(t \leq T \leq t + h | T \geq t) \approx \lambda(t)h$. Therefore, the hazard function is fundamental in determining the conditional probability of pavement failure.

The hazard rate function is also of common use in reliability analysis since it can be related to the probability that the pavement structure will suffer failure in a small time interval. From (3) we have that,

$$\lambda(t) = \lim_{h \to 0} \frac{\left[\frac{P(t \leq T \leq t + h)}{P(T \geq t)}\right]}{h} = \lim_{h \to 0} \frac{\left[\frac{F(t + h) - F(t)}{S(t)}\right]}{h} = \lim_{h \to 0} \frac{F(t + h) - F(t)}{h} \cdot \frac{1}{S(t)} = \frac{f(t)}{S(t)} \qquad (2)$$

Note from (2) that $\int_0^t \lambda(u)du = \int_0^t \frac{f(u)}{S(u)} du$, and therefore $S(t) = \exp\left(-\int_0^t \lambda(u)du\right)$.

Because of the interpretation of the hazard rate function as the instantaneous conditional probability of pavement failure, the following analysis will be based on the modeling of this function. Furthermore, it has been previously shown that the hazard rate function is directly related to the survival function, the unconditional failure function, and the cumulative failure function. Therefore, it is straightforward to move from one function to the next.

2.3 Modeling Hazard Rate with Right Censored Data

Because of the different types of failures that can occur in a pavement structure, and the heterogeneity of factors involved in the performance of the pavement structure, it could be disputed that pavement structures can exhibit both increasing and decreasing failure rates (Prozzi *et al.*, 2000). However, we can expect that overall hazard rate should increase with age: the instantaneous probability of failure of a given pavement structure increases as the time that the pavement has been in service increases.

A Weibull distribution has been proven to be effective in cases when the hazard rate is variable (Prozzi *et al.*, 2000). The hazard function for a Weibull regression model accounting for time invariant exogenous variables is given by,

$$\lambda(t, \boldsymbol{X}, \boldsymbol{\beta}, \sigma) = \frac{1}{\sigma}(t)^{\frac{1-\sigma}{\sigma}} \exp(\boldsymbol{X\beta})^{-\frac{1}{\sigma}} \tag{3}$$

where, $\boldsymbol{X}$ is the set of explanatory variables, and $\boldsymbol{\beta}, \boldsymbol{\sigma}$ are the set of parameters to be estimated. By re-arranging the terms on (5) the hazard function for the Weibull model can be expressed in the accelerated failure time form as follows,

$$\lambda(t, \boldsymbol{X}, \boldsymbol{\beta}, \sigma) = \frac{1}{\sigma}[t \exp(-\boldsymbol{X\beta})]^{\frac{1-\sigma}{\sigma}} \exp(-\boldsymbol{X\beta}) \tag{4}$$

Based on the accelerated failure time form of the Weibull model, the density function and the survival function can be expressed as,

$$f(t, \boldsymbol{X}, \boldsymbol{\beta}, \sigma) = \frac{1}{\sigma}(t)^{\frac{1-\sigma}{\sigma}} \exp(\boldsymbol{X\beta}) \left\{\exp\left[-\exp(\boldsymbol{X\beta})\,(t)^{\frac{1}{\sigma}}\right]\right\} \tag{5}$$

$$S(t, \boldsymbol{X}, \boldsymbol{\beta}, \sigma) = \exp\left\{-(t)^{\frac{1}{\sigma}} \exp\left[\left(-\frac{1}{\sigma}\right)(\boldsymbol{X\beta})\right]\right\} \tag{6}$$

Estimation of the parameters of interest $(\boldsymbol{\beta}, \sigma)$ can be done by means of maximum likelihood estimation. First define a dummy variable $c = 0$ if a given pavement structure is right censored at time t and 1 otherwise. If the time to failure observation of a pavement structure is uncensored, then the conditional density of $t = T$ is $f(t, \boldsymbol{X}, \boldsymbol{\beta}, \sigma)$. Additionally, the probability that t is censored is $P(t \geq t_f | \boldsymbol{X}, \boldsymbol{\beta}, \sigma) = 1 - F(t, \boldsymbol{X}, \boldsymbol{\beta}, \sigma) = S(t, \boldsymbol{X}, \boldsymbol{\beta}, \sigma)$ where t_f is the time at which data collection ended. Then the conditional likelihood function for pavement section i is given by $\ell_i(\boldsymbol{\beta}, \sigma) = f(t_i, \boldsymbol{x_i}, \boldsymbol{\beta}, \sigma)^{c_i} S(t_i, \boldsymbol{x_i}, \boldsymbol{\beta}, \sigma)^{1-c_i}$.

Because the different pavement sections are assumed to be independent, the likelihood function is the product over i,

$$\mathcal{L}(\boldsymbol{\beta}, \sigma) = \prod_{i=1}^{n} f(t_i, \boldsymbol{x_i}, \boldsymbol{\beta}, \sigma)^{c_i} S(t_i, \boldsymbol{x_i}, \boldsymbol{\beta}, \sigma)^{1-c_i} \tag{7}$$

Then, for computational simplicity, the parameters of interest can be obtained by maximizing the log-likelihood function,

$$\log \mathcal{L}(\boldsymbol{\beta}, \sigma) = \sum_{i=1}^{n} c_i f(t_i, \boldsymbol{x_i}, \boldsymbol{\beta}, \sigma) + (1 - c_i) S(t_i, \boldsymbol{x_i}, \boldsymbol{\beta}, \sigma) \tag{8}$$

2.4 Semi-Parametric Approach

The previous method of performing of the survival analysis is parametric in the sense that the distribution of the density function for time to failure of a pavement section had to be specified. One might argue that making this assumption is restrictive. An alternate approach that is commonly used is performing a semi-parametric analysis which does not require any assumption on the distribution of failure times.

A form of semi-parametric regression model that is useful in this analysis is,

$$\lambda(t, \boldsymbol{X}, \boldsymbol{\beta}) = \lambda_0(t) r(\boldsymbol{X}, \boldsymbol{\beta}) \tag{9}$$

The hazard function shown in (9) is the product of two different functions. The first component describes how the hazard rate changes with time, while the second component captures the effect of exogenous variables on failure rate. When the $r(\boldsymbol{X}, \boldsymbol{\beta})$ function is parameterized such that $r(\boldsymbol{X} = \boldsymbol{0}, \boldsymbol{\beta}) = 1$, then $\lambda_0(t)$ is described as the baseline hazard function (Hosmer *et al.*, 2008). Under this model specification, the ratio of the hazard functions of two different pavement structures is proportional and depends only on $r(\boldsymbol{X}, \boldsymbol{\beta})$,

$$HR(t, \boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{\beta}) = \frac{\lambda(t, \boldsymbol{x_1}, \boldsymbol{\beta})}{\lambda(t, \boldsymbol{x_2}, \boldsymbol{\beta})} = \frac{\lambda_0(t) r(\boldsymbol{x_1}, \boldsymbol{\beta})}{\lambda_0(t) r(\boldsymbol{x_2}, \boldsymbol{\beta})} = \frac{r(\boldsymbol{x_1}, \boldsymbol{\beta})}{r(\boldsymbol{x_2}, \boldsymbol{\beta})} \tag{10}$$

If a parameterization of $r(\boldsymbol{X}, \boldsymbol{\beta}) = \exp(\boldsymbol{X}\boldsymbol{\beta})$ is used, the model is referred to as a proportional hazards model. Additionally, using calculus, it can be shown that $f(t, \boldsymbol{X}, \boldsymbol{\beta}) = \lambda(t, \boldsymbol{X}, \boldsymbol{\beta}) \cdot S(t, \boldsymbol{X}, \boldsymbol{\beta})$ (Hosmer *et al.*, 2008). Consequently, estimating $\boldsymbol{\beta}$ using a log-likelihood function equivalent to (8) cannot be done since the form of $\lambda_0(t)$ is not specified. Cox (1972) proposed using a "partial likelihood function" to estimate the parameters of interest. The partial likelihood function is given by,

$$\mathcal{L}_p(\boldsymbol{\beta}) = \prod_{i=1}^{n} \left[\frac{\exp(\boldsymbol{x_i}\boldsymbol{\beta})}{\sum_{j \in R(t_i)} \exp(\boldsymbol{x_j}\boldsymbol{\beta})} \right]^{c_i} \tag{11}$$

where the summation in the denominator is over all the pavement sections in the risk set at time t_i. The pavement sections at risk at time t_i are all those that at time t_i have neither failed or been censored. Additionally, note that censored observations are not included in the numerator, but their instantaneous probabilities of failure are appropriately accounted for in the denominator.

Finally, the log-likelihood function is,

$$\log \mathcal{L}_p(\boldsymbol{\beta}) = \sum_{i=1}^{n} \left\{ \boldsymbol{x_i}\boldsymbol{\beta} - \log \left[\sum_{j \in R(t_i)} \exp(\boldsymbol{x_j}\boldsymbol{\beta}) \right] \right\} \tag{12}$$

It has to be noted that by using the semi-parametric approach the restrictiveness of the model is reduced, however the use of the model in predicting time to failure of a pavement structure is limited since the baseline hazard rate has remained unspecified.

3 SURVIVAL ANALYSIS DATASET

For the following analysis, the authors have limited the possible failure mode to rutting. In survival analysis it is extremely important to appropriately define pavement failure based on the selected failure mode. The failure threshold has been defined as 0.5 in (12.5 mm). This threshold closely corresponds to the overall failure criteria used in the MEPDG.

The Long Term Pavement Performance (LTPP) database Standard Data Release 23.0 was used for identifying flexible pavement sections to be included in the current analysis. The analysis is limited to flexible pavement sections that consist of an asphalt layer on top of an untreated granular base. For specific information on the individual SPS-1 sections please refer to http://www.ltpp-products.com/.

The Specific Pavement Sections (SPS) were primarily identified because they were constructed after the LTPP monitoring program started and therefore a complete monitoring of these pavement sections has been performed through their entire service lives. In the case of SPS sections, $t = 0$ occurs at the time that the pavement section is opened to traffic after the initial construction.

Additionally, flexible pavement sections from the General Pavement Studies were also selected if a major rehabilitation/reconstruction occurred after the beginning of the LTPP program. The authors have made the assumption that these sections that have been reconstructed should perform similarly to new constructions. Consequently, the GPS sections that match these criteria have been included in the analysis from the time of the reconstruction, and therefore $t = 0$ occurs at the time that the pavement section is opened to traffic after the major rehabilitation/reconstruction. Based on the previous criteria 69 flexible pavement sections were identified containing the required information for a survival analysis based on rutting.

Figure 1 displays the measured survival probability function for pavement failure due to rutting (field observation based on LTPP). As expected, more failures due to rutting seem to occur earlier in the life of the pavement structures. After 8.5 years, failures due to rutting are infrequent. It has to be cleared that the current analysis only focuses on rutting and therefore the previous behavior is expected. However, failures due to fatigue cracking should become the predominant failure mode at this stage in the service life of the pavement structure.

The previous behavior is confirmed in Figure 2 (also based actual field data). Before approximately 8 years into the service life of the pavement structure the hazard rate steadily increases indicating a relatively uniform failure rate due to rutting during the initial years of service life of the pavement structure.

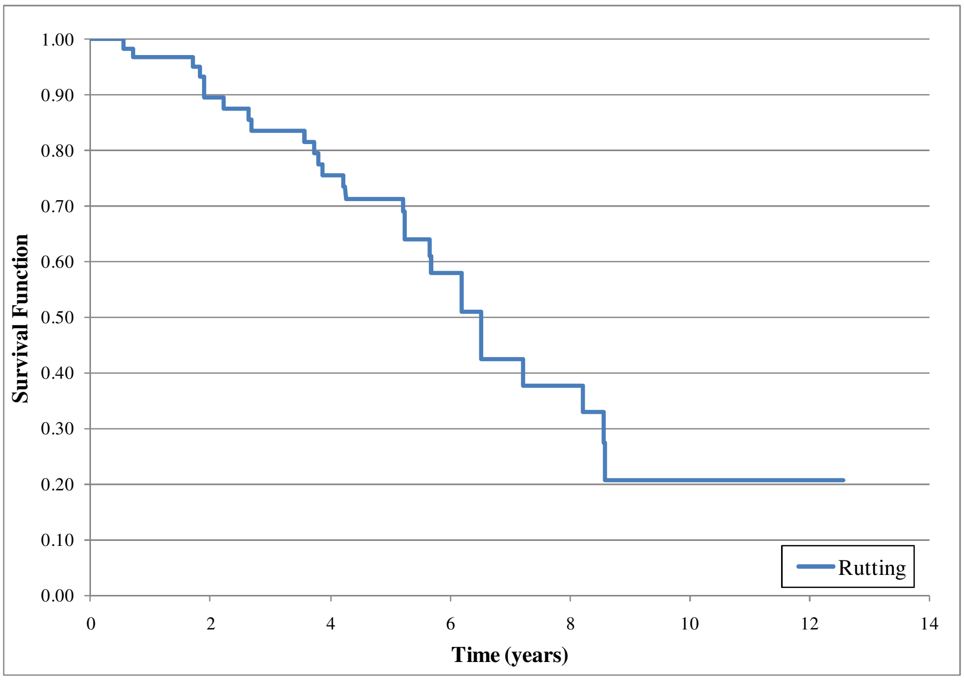

Figure 1: Observed survival function for SPS-1 sections.

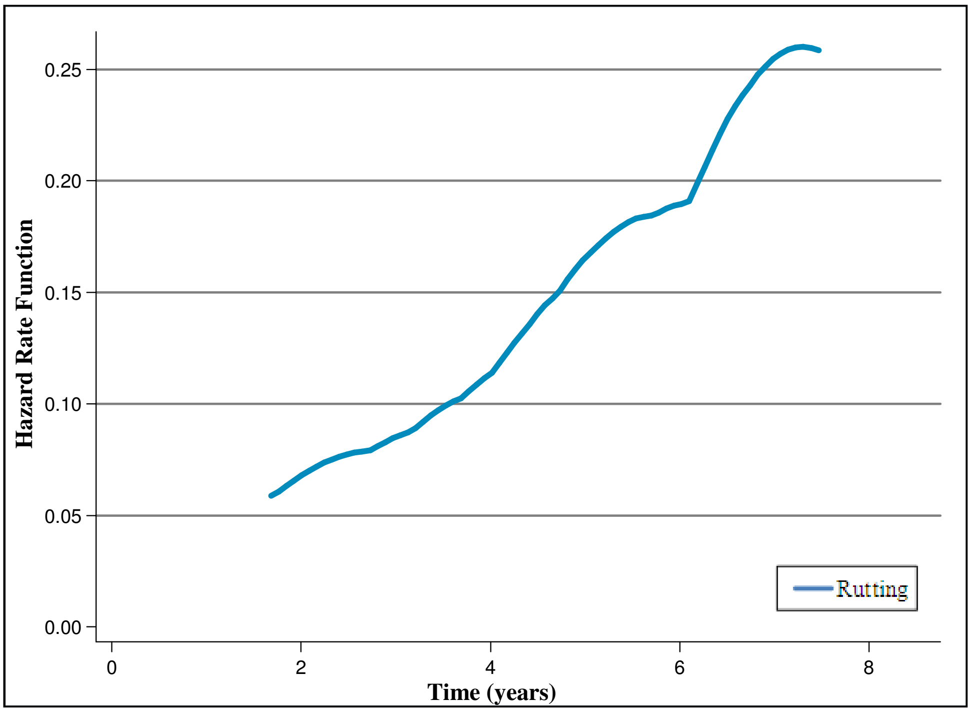

Figure 2: Observed hazard rate functions for SPS-1 sections.

3.1 Selected Exogenous Variables

There are many factors that influence pavement rutting resistance, and consequently pavement failure due to rutting. In an effort to capture the effect of these exogenous factors, the authors have selected several traffic, structural, environmental, and material properties that have been considered to be relevant in pavement performance. The chosen variables are shown in Table 1.

Table 1: Variables Considered in the Analysis

Category	Variable	Abbreviation	Units
Structural	Total thickness of AC layers	Thickness AC	in
	Total thickness of base layer	Thickness GB	in
Material	Asphalt binder content	AC	%
	Air void content	AV	%
Traffic	Equivalent Single Axle Loads (x 1000)	kESAL	#
Environmental	Average total annual precipitation	Total Precip.	in
	Average number of days for which precipitation > 0.01 in per year	Wet Days	#
	Average total annual snowfall	Total Snow	in
	Average number of days with snow cover per year	Snow Days	#
	Average number of days above 89°F	Days > 90°F	#
	Average number of days below 32°F	Days < 32°F	#
	Average freezing index	FI	°F-days

The previously selected pavement and environment characterization information was extracted from the LTPP database from the Administration, Climate_Summary_Data, Material_Test, Traffic, and Monitoring modules. The variable selection additionally corresponds with some of the variables that were identified to be statistically significant by Wang *et al.* (2005) on their survival analysis of fatigue cracking data.

4 SURVIVAL ANALYSIS MODEL ESTIMATION

4.1 Parametric Approach

Table 2 shows the rutting survival model estimates. Several of the environmental variables (average total annual precipitation, average number of days with precipitation, average total annual snowfall, average number of days with snow cover per year, and average number of days above 90°F) have been excluded from the model since they were found to be not statistically significant and highly correlated among themselves. By removing the previous variables an improvement in model fit and remaining exogenous variables statistical significance was achieved. The log-likelihood for the parametric model is -40.86.

Table 2: Weibull Parametric Regression for Pavement Failure due to Rutting

Variable	Coefficient	Std. Deviation	χ^2	p-value	Hazard Ratio
Intercept	4.7326	0.7852	36.32	0.0000	-
kESALs	-0.0017	0.0006	8.34	0.0039	1.0027
Thickness AC	0.0795	0.0494	2.60	0.1072	0.8802
Thickness GB	-0.0454	0.0289	2.48	0.1155	1.0756
AC	-0.2647	0.0880	9.06	0.0026	1.5294
AV	-0.0646	0.0365	3.14	0.0766	1.1093
Days < 32.0°F	-0.0084	0.0028	8.95	0.0028	1.0136
FI	0.0004	0.0003	1.12	0.2893	0.9994
σ	0.4484	0.0679	-	-	-

The most significant variables in the model are traffic as measured in kESALs, asphalt binder content, air void content, and the number of days that the temperature is below 32.0°F. Because the hazard and survival models are non-linear, a direct interpretation of the coefficients is not straightforward. Consequently, Table 2 also includes the hazard ratio which basically represents the increase (or decrease) on the hazard rate due to a unit increase of the exogenous variable. In the case of rutting, the asphalt binder content has the biggest effect on the hazard rate: a 1% increase in asphalt content corresponds to a 53% increase to the hazard rate. Also, a 1 in. increase in the asphalt layer thickness can be related to a 12% decrease in the hazard rate, while a 1% increase in the air void content of the asphalt mix can be associated with a 11% increase in the hazard ratio. The previous effects on the hazard ratio and failure rate are as expected. Additionally, the thickness of the granular base was also found to be important, but not as expected: a 1 in. increase in granular base thickness can be correlated to an 8% increase in hazard rate.

4.2 Semi-Parametric Approach

The results from the proportional hazard approach follow a similar trend to the previous results. As before, Table 3 shows the results from the proportional hazard rutting model. As in the parametric approach, the same environmental variables were removed from the model due to lack of statistical significance. The log-likelihood for the parametric model is -83.31.

Table 3: Proportional Hazards Regression for Pavement Failure due to Rutting

Variable	**Coefficient**	**Std. Deviation**	χ^2	**p-value**	**Hazard Ratio**
kESALs	0.0037	0.0014	6.84	0.0089	1.0040
Thickness AC	-0.1757	0.1160	2.29	0.1301	0.8390
Thickness GB	0.1147	0.0684	2.81	0.0937	1.1220
AC	0.5015	0.2107	5.67	0.0173	1.6510
AV	0.1590	0.0849	3.51	0.0609	1.1720
Days < 32.0°F	0.0193	0.0068	7.99	0.0047	1.0200
FI	-0.0010	0.0008	1.74	0.1876	0.9990

As in the parametric model, the most significant variables are traffic as measured in kESALs, asphalt binder content, air void content, and the number of days that the temperature is below 32.0°F. As in the previous model estimation, the asphalt binder content has the biggest effect on the hazard rate (a 1% increase in asphalt content corresponds to a 65% increase to the hazard rate. Furthermore, a 1 in. increase in the asphalt layer thickness can be related to a 16% decrease in the hazard rate, while a 1% increase in the air void content of the asphalt mix can be associated with a 17% increase in the hazard ratio. Additionally, a 1 in. increase in granular base thickness can be related to a 12% increase in hazard rate.

5 CONCLUSIONS

This paper presents a framework for determining time to failure of pavement structures and their associated probability of survival, associated with rutting under several environmental conditions and diverse structural and material properties. An application for the analysis was performed using GPS and SPS data from the LTPP database. As expected the results indicate that traffic, asphalt layer thickness and the asphalt mix properties (asphalt and air void content) have a significant effect of the probability that a pavement structure performs appropriately for a given number of years. The number of days a year that a pavement structure is below 32.0°F was also found to be highly significant in the model, although its overall effect on the hazard rate is not as important.

The authors believe that the approach presented here will be valuable, at a state or even at a district level, because the model is able to capture with more precision what factors cause pavement structures to fail due to rutting at a faster rate. This is given by the hazard function.

State agencies or District Offices can use their Pavement Management System (PMS) information to update these models and, thus, increasing considerably the reliability of the model estimates. This can be extremely helpful for administrating a local pavement network and to prioritize maintenance and rehabilitation activities.

Additionally, survival models have considerable advantages over commonly used Markov Models in pavement management since the models can make use of all available PMS data and more importantly are continuous, which is how pavement deterioration evolves in the field. Markov models, on the other hand, require a discretization of the failure function, and additionally make the incorrect assumption that the current state of a pavement function is unrelated to the previous performance of the pavement structure.

Finally, the authors found that although the semi-parametric approach produced higher likelihood values (more than two times that of the parametric model), the hazard ratios estimated by the semi-parametric and the parametric approaches are in most cases not statistically different. This indicates that the Weibull model assumption for the parametric hazard function is appropriate in this case. This finding is important since the semi-parametric approach is computationally simpler and can be swiftly estimated in cases when no prediction of time to failure is required.

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