Thermal Cracking Simulations of Aged Asphalt Pavements using Viscoelastic Functionally Graded Finite Elements

E. V. Dave, W. G. Buttlar & G. H. Paulino
Department of Civil and Environmental Engineering, University of Illinois, Urbana, Illinois, United States

ABSTRACT: Low temperature or thermal cracking is a major source of asphalt pavement deterioration in cold climates and/or in regions where high cooling rates are present. As asphalt pavements age and become stiffer, the potential for thermally induced cracks which initiate from the pavement surface increases. The current state of practice in the simulation of thermal cracking for design and analysis does not account for the presence of property gradients, which have been shown to exist in asphalt pavement surface courses as a result of temperature- and aging-induced gradients. These gradients need to be considered in order to better understand how they may affect the initiation and propagation of thermal cracks. A novel simulation technique utilizing finite elements with viscoelastic graded elements enable accurate and efficient simulation of viscoelastic functionally graded materials (VFGM) such as those present in aged asphalt pavements. In the current study, aging and temperature induced property gradients in asphalt pavements are simulated in the context of low temperature cracking. Three pavement structures are simulated along with three mixture types. Cooling events with different peak cooling rates have been simulated for each of these pavement systems. Simulation results for VFGM cases are compared with results obtained using homogeneous material properties as a function of depth in the pavement. The results demonstrate significantly different thermal cracking predictions when aging gradients are considered, as the age stiffening significantly increases stress concentrations near the surface of the simulated pavements. The effect of pavement aging examined in this study provided new insight towards the mechanisms of thermal cracking in asphalt pavements. The techniques introduced can be readily adapted in existing thermal cracking models which utilize linear viscoelastic material properties.

KEY WORDS: Thermal cracking, asphalt concrete, aging, functionally graded materials, finite elements, viscoelasticity.

1 INTRODUCTION AND MOTIVATION

Thermal cracking or low temperature cracking of asphalt concrete pavements continues to be a leading cause of premature pavement deterioration in regions of cold climate and/or where significant thermal cycling occurs. Recent advances in fracture testing and modeling of asphalt concrete materials have greatly aided in the understanding of the key mechanisms behind this important pavement distress, which can greatly reduce pavement lifespan and the lifespan of subsequent rehabilitation cycles. Integrated laboratory testing and cracking simulation procedures have been successfully implemented for thermal cracking predictions (Marasteanu et al. 2007, Dave et al. 2008). However, previous studies on thermal cracking ignore age stiffening of asphalt concrete and corresponding property gradients across the pavement thickness.

The constituents of asphalt concrete include asphalt binder (bitumen) and mineral aggregates. Asphalt binder is derived from crude oil as a by-product of fractional distillation. Due to its organic nature, asphalt binder undergoes oxidative aging as time progresses, the effect of which is most easily identified as hardening or stiffening of the binder, and subsequently, the mixture. The effect of aging creates graded material properties due to variation in the amount of aging through the depth of the pavement. The Strategic Highway Research Program (SHRP) Project A-368 investigated chemical composition changes occurring during binder aging. The final report from this project identifies the process of age hardening as a non-reversible and continuous process that extends throughout the life of a pavement (Branthaver et al. 1993). The aging and temperature induced property gradients have been well documented by several researchers in the field of asphalt pavements (for example, Mirza and Witczak 1996 and Chiasson et al. 2008). The effects of aging are most pronounced at the surface of the asphalt layer. In general, thermal cracks initiate and propagate from the surface of pavement and progress downwards. Hence, aging gradients are expected to have a detrimental effect on thermal cracking distress, particularly with respect to crack initiation. Ignoring aging gradients in prediction models can lead to the significant under prediction of a pavement's thermal cracking potential. The accuracy and efficiency of functionally graded approaches in capturing aging related property gradients have been previously demonstrated in the context of pavement stress responses under tire loading (Buttlar et al. 2005, Dave et al. 2010).

This paper describes the development and application of a specialized finite element method that is suited for simulations of viscoelastic functionally graded materials (VFGM), such as asphalt pavements that have property gradients caused by aging and temperature gradients. The formulation is briefly described followed by results from verification studies and application examples involving thermal cracking simulations in flexible pavements, with property gradients as predicted by the Mechanistic Empirical Pavement Design Guide software.

2 VISCOELASTIC FUNCTIONALLY GRADED FINITE ELEMENT METHOD

This section presents the formulation developed for VFGM finite element implementation. The subsections describe general functionally graded viscoelastic theory, details of the finite element formulations, and selected model verification examples.

2.1 Viscoelastic Theory

General viscoelastic theory can be found in several textbooks and articles, for example, Christensen (1982). A generalized Maxwell model is utilized in this study due to its flexibility in representing a wide variety of viscoelastic materials as well as the availability of established formulations in the literature. The constitutive relationship for generalized Maxwell model can be given as,

$$\boldsymbol{\sigma}(\mathbf{x},\xi)=\mathbf{E}_{\infty}(\mathbf{x})\boldsymbol{\varepsilon}(\mathbf{x},\xi)+\int_{0}^{t}\mathbf{E}_{\mathbf{t}}\left(\mathbf{x},\xi-\xi'\right)\frac{d\boldsymbol{\varepsilon}\left(\mathbf{x},\xi'\right)}{d\xi'}d\xi' \tag{1}$$

where $\boldsymbol{\sigma}$ is stress, $\boldsymbol{\varepsilon}$ is strain, ξ is reduced time and $\mathbf{E}_{\infty}$ is fully relaxed modulus and $\mathbf{E}_{\mathbf{t}}$ is relaxation modulus for the Maxwell chains. The relaxation modulus for Maxwell units is given by,

$$\mathbf{E}_t = \sum_{m=1}^{M} \mathbf{E}_m(\mathbf{x}) e^{-\left(\xi-\xi'\right)/\boldsymbol{\tau}_m(\mathbf{x})};\ \boldsymbol{\tau}_m(\mathbf{x}) = \frac{\boldsymbol{\eta}_m(\mathbf{x})}{\mathbf{E}_m(\mathbf{x})} \tag{2}$$

The material parameters $\mathbf{E}_m, \boldsymbol{\eta}_m$ are spring coefficients and viscosities for the m^{th} Maxwell unit. The spring coefficients and viscosities are related through relaxation times $\boldsymbol{\tau}_m$ and the total number of Maxwell units in the model is given by M. The effect of temperature on the material properties is accounted for through use of time-temperature superposition principle. The superposition is governed time-temperature shift factor a_T which is a material property. The real time t is related to the reduced time ξ and temperature T as,

$$\xi = \int_0^t \frac{dt'}{a_T\left(T, t'\right)} \tag{3}$$

For isotropic conditions the above shown constitutive relationships can be re-written in form of deviatoric and volumetric stress-strain relationships as,

$$\begin{aligned} \sigma_{kk}(\xi) &= 3K_\infty(\mathbf{x})\varepsilon_{kk}(\xi) + \int_0^\xi 3K_t\left(\mathbf{x}, \xi-\xi'\right)\frac{d\varepsilon_{kk}\left(\xi'\right)}{d\xi'}d\xi' \\ s_{ij}(\xi) &= 2G_\infty(\mathbf{x})\varepsilon_{ij}^s(\xi) + \int_0^\xi 2G_t\left(\mathbf{x}, \xi-\xi'\right)\frac{d\varepsilon_{ij}^s}{d\xi'}d\xi' \end{aligned} \tag{4}$$

where, $K_\infty, K_t, G_\infty, G_t$ are shear and bulk relaxation modulus components following the similar descriptions as shown before. The deviatoric strain components are shown by ε_{ij}^s and the corresponding stress components by s_{ij}, these are evaluated as,

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij};\ \varepsilon_{ij}^s = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij} \tag{5}$$

where, δ_{ij} is Kronecker's delta.

2.2 VFGM Finite Elements

The VFGM finite elements utilized in this study are based on the formulations developed by Dave et al. (2010). The time-integration approach in this formulation is based on the recursive incremental scheme developed by Yi and Hilton (1994). Similar schemes have been utilized for solving viscoelastic finite element problems by several researchers (for example, Muliana and Khan 2008). In field of asphalt concrete an incremental-recursive scheme has been utilized by Dai and You (2009) for analysis of asphalt mixtures undergoing damage in lab sized specimens.

The incremental-recursive formulations (Zocher et al. 1997) rely on determination of incremental stress components $(d\boldsymbol{\sigma})$ in response to the strain increment $(d\boldsymbol{\varepsilon})$ given by,

$$d\boldsymbol{\sigma}(\xi) = \mathbf{K}(\mathbf{x}, \xi) \times d\boldsymbol{\varepsilon}(\xi) + d\boldsymbol{\sigma}^R(\xi) \tag{6}$$

where, the stiffness is given by $\mathbf{K}$ and the viscoelastic history effect is accounted through residual stress term $d\boldsymbol{\sigma}^R$. Using the constitutive relationships shown in equation (4) and recursive-incremental formulation in equation (6), the volumetric and deviatoric stress increments can be evaluated as,

$$\begin{aligned} ds_{ij}(\xi) &= 2\left[G_\infty(\mathbf{x}) + \sum_{m=1}^{M}\frac{G_m(\mathbf{x})\tau_m(\mathbf{x})}{d\xi}\left(1-e^{-d\xi/\tau_m(\mathbf{x})}\right)\right]d\varepsilon_{ij}^s(\xi) + ds_{ij}^R(\xi) \\ d\sigma_{kk}(\xi) &= 3\left[K_\infty(\mathbf{x}) + \sum_{m=1}^{M}\frac{K_m(\mathbf{x})\tau_m(\mathbf{x})}{d\xi}\left(1-e^{-d\xi/\tau_m(\mathbf{x})}\right)\right]d\varepsilon_{kk}(\xi) + d\sigma_{kk}^R(\xi) \end{aligned} \tag{7}$$

At any reduced time ξ_n the increment in reduced time $(d\xi)$ and the corresponding strain rates $(\mathbf{R})$ can be approximated as,

$$d\xi \approx \Delta\xi = \xi_n - \xi_{n-1};\ \mathbf{R} = \frac{d\boldsymbol{\varepsilon}}{d\xi} \approx \frac{\Delta\boldsymbol{\varepsilon}}{\Delta\xi}. \tag{8}$$

The residual stress can be evaluated for deviatoric and volumetric components using the approximations shown in equation (8) as,

$$ds_{ij}^{R}\left(\xi_n\right) = \sum_{m=1}^{M} -(1-e^{-\Delta\xi/\tau_m(\mathbf{x})})S_m\left(\xi_n\right);\ d\sigma_{kk}^{R}\left(\xi_n\right) = \sum_{m=1}^{M} -(1-e^{-\Delta\xi/\tau_m(\mathbf{x})})V_m\left(\xi_n\right) \tag{9}$$

where, symbols S_m and V_m represent viscoelastic (history) stress contributions at any given reduced time. These effects account for hereditary contributions should be tracked for each stress component throughout the entire range of time-steps used in a given simulation. Also notice that these terms are independent for each Maxwell unit in the material constitutive properties. The viscoelastic stress contributions are updated for each time increment. Using the approximate strain rate (equation (8)) and the expansion of equations (4) and (7) the viscoelastic stress contributions can be evaluated as,

$$\begin{aligned} S_m\left(\xi_n\right) &= 2G_m\left(\mathbf{x}\right)\tau_m\left(\mathbf{x}\right)\left(1-e^{-\Delta\xi/\tau_m(\mathbf{x})}\right)R_{ij}^{s} + S_m\left(\xi_{n-1}\right)e^{-\Delta\xi/\tau_m(\mathbf{x})} \\ V_m\left(\xi_n\right) &= 3K_m\left(\mathbf{x}\right)\tau_m\left(\mathbf{x}\right)\left(1-e^{-\Delta\xi/\tau_m(\mathbf{x})}\right)R_{kk} + V_m\left(\xi_{n-1}\right)e^{-\Delta\xi/\tau_m(\mathbf{x})} \end{aligned} \tag{10}$$

Notice that in the above shown formulations, the material properties $\left(G_\infty, K_\infty, G_m, K_m, \tau_m\right)$ are a function of space $(\mathbf{x})$, making the formulation non-homogeneous in nature. In order to create a smooth gradation of material properties, which is expected in the case of aged asphalt concrete pavement, it is necessary to capture the material gradation effect within the element. These types of specialized elements are commonly referred to as “graded elements”. In general, two approaches have been proposed for graded elements: (a) direct sampling of material properties as Gauss points; and (b) Graded elements using generalized iso-parametric formulations (Kim and Paulino 2002). Dave et al. (2010) extended the generalized iso-parametric formulations for viscoelastic finite elements. Buttlar et al. (2006) have utilized elastic graded elements for the analysis of aged asphalt pavements. In the present paper, the material gradation within the element is accounted for through direct sampling of material properties at the Gauss points.

2.3 Implementation and Verification

The VFGM finite element formulations discussed in the previous section has been implemented in the commercial software ABAQUS by means of a user defined material subroutine (UMAT). Commercial software is chosen to take benefit of the existing features in the code and to ensure computational efficiency. In order to verify the formulations and their successful implementation, a series of verification examples were conducted. The verifications were conducted in three stages, namely viscoelastic verifications, thermo-viscoelastic verifications, and VFGM verifications. The verifications were conducted by comparing the response of VFGM simulations to analytical reference solutions.

The viscoelastic verifications were first conducted using creep and relaxation behavior of homogeneous finite elements at a selected reference temperature to imposed traction and boundary conditions. The material properties utilized for the viscoelastic and thermo-viscoelastic problems are shown in Figure 1. Notice that the time-temperature superposition in these examples is performed through use of WLF equations (Christensen, 1982). Figure 2 shows the creep and relaxation response of the boundary value problems in the form of creep displacement and stress relaxations. Identical results were obtained for analytical solutions and the VFGM finite elements developed in this study. This demonstrates the veracity of the formulations and their implementation.

Thermo-viscoelastic verifications were performed to verify the accuracy of the VFGM element formulations in the context of time dependent temperature conditions and temperature dependent viscoelastic properties. The boundary value problem simulated in this case is similar to the thermal stress restrained specimen test (TSRST), which is sometimes used for the evaluation of thermal cracking performance of asphalt concrete (AASHTO TP-10). In order to ensure good accuracy for thermal cooling and warming conditions, the temperature boundary conditions were chosen to impose both warming and cooling events. The results from the VFGM elements proposed in this study were compared with the results obtained from the commercial software *ABAQUS*. The commercial software does not have graded element capability hence the problem was solved using homogenous material properties. Figure 3 shows the variation of temperature with time as well as the corresponding thermal stresses generated in the restrained viscoelastic body. The stress response is shown for the formulations and implementation from the present study as well as those obtained using *ABAQUS*, showing excellent agreement.

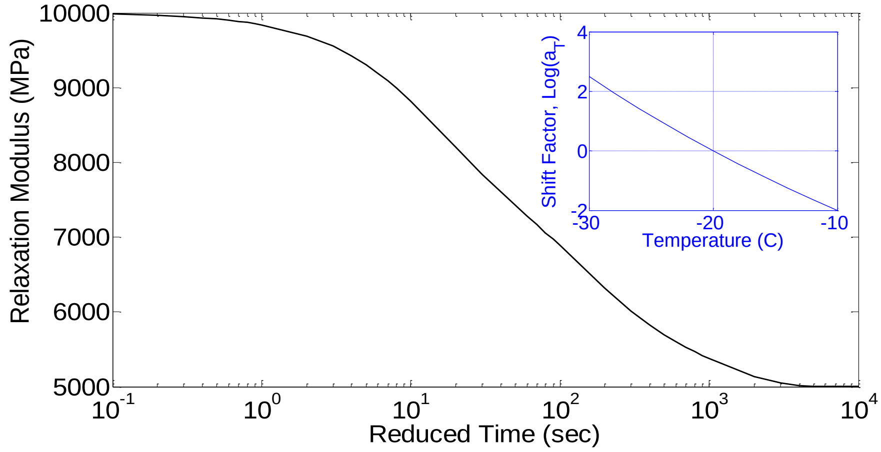


Figure 1: Relaxation modulus mastercurve utilized for verification examples. (insert: Time-temperature superposition shift-factors, WLF equation).

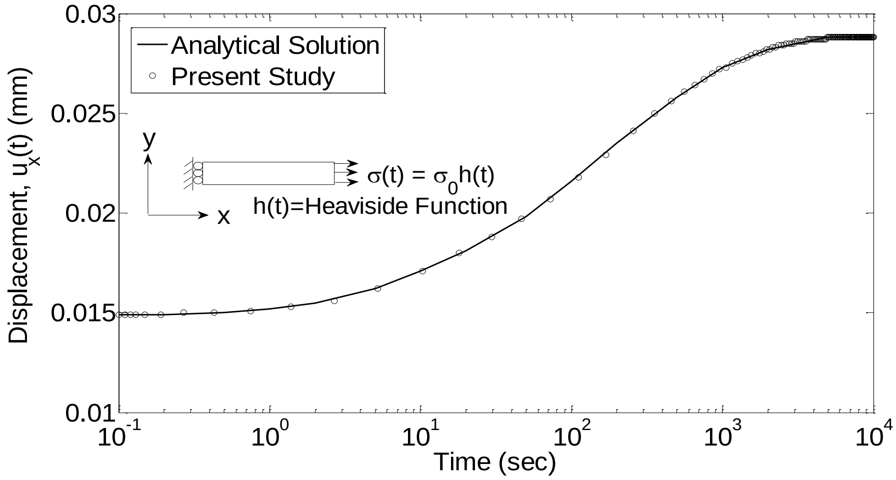


Figure 2a: Comparison of analytical and finite element solution for viscoelastic bar in creep loading conditions.

The final verification example presented in this paper is for a VFGM problem. This is an important verification step since it ensures the accuracy of present approach for simulation of non-homogeneous viscoelastic problems. Analysis is conducted for the creep response of an exponentially graded VFGM bar. The results from the present study are compared with an analytical solution available in the literature (Mukherjee and Paulino 2003) for the problem shown in Figure 4. Based upon the excellent agreement observed, the VFGM model developed and implemented in this study was considered to be successfully verified.

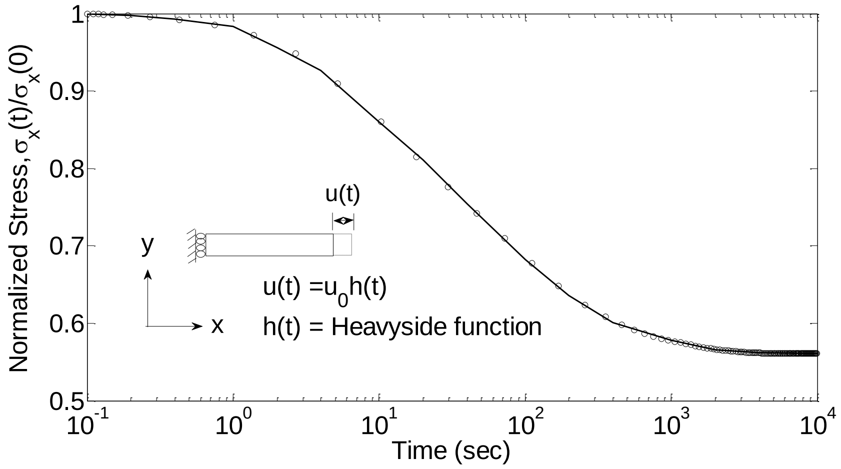


Figure 2b: Comparison of analytical and finite element solution for viscoelastic bar in fixed grip loading conditions (stress relaxation).

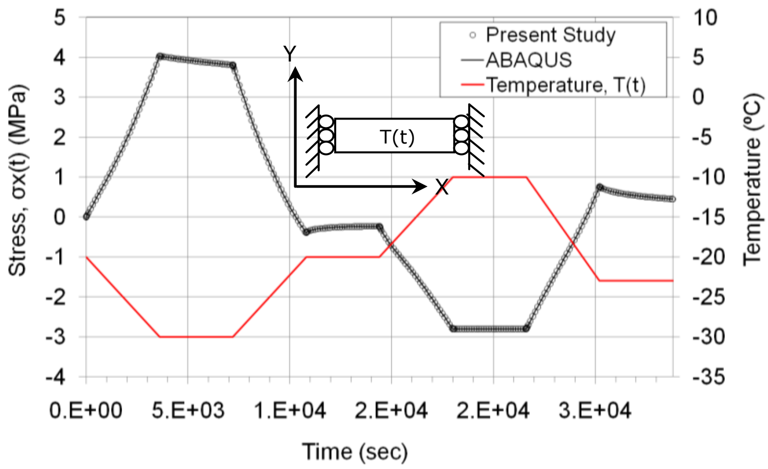


Figure 3: Comparison of finite element solutions from present study and commercial software *ABAQUS* for thermal restrained boundary value problem.

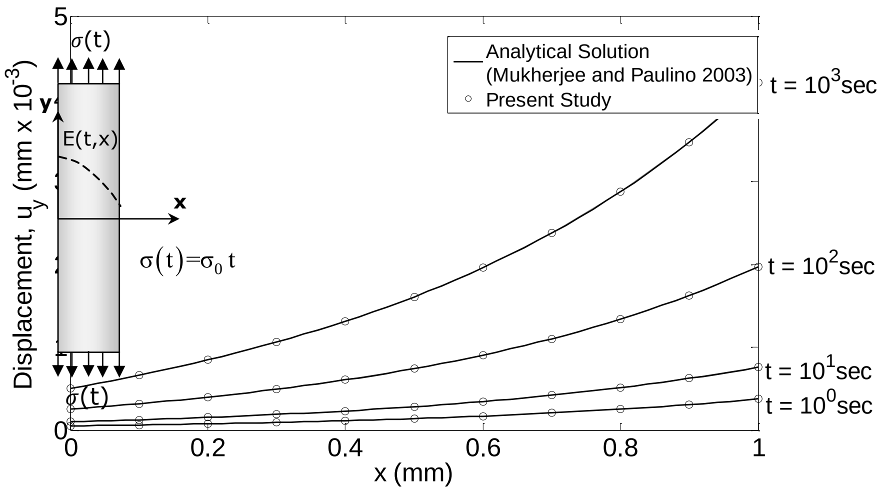


Figure 4: Comparison of creep displacement results evaluated using analytical solution and finite element method (present study) for an exponentially graded VFGM bar.

3 SIMULATION MODEL AND RESULTS

3.1 Pavement Sections, Materials and Cracking Model

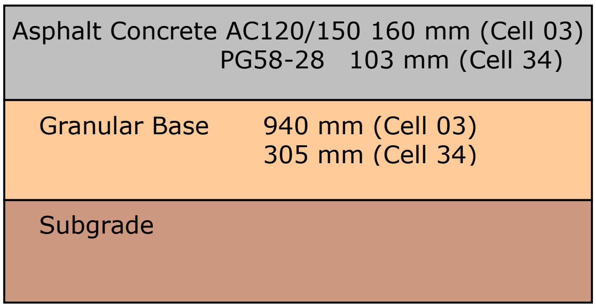
In the present study, two pavement sections from Minnesota Department of Transportation's Road Research Facility (MnROAD) are studied, namely, MnROAD cell 03 and cell 34. The schematics of the pavement sections are shown in Figure 6a. Fracture and thermal volumetric expansion and contraction properties for these sections are reported by Marasteanu et al. (2006). The global aging model (GAM) by Mirza and Witczak (1996) was utilized for determining the graded viscoelastic properties. Complex modulus (E*) is predicted by GAM as a function of depth of asphalt concrete layer, the same model was utilized for determined unaged material properties. Notice that the unaged predictions made in GAM are based on the "Witczak Predictive Equation", which is also utilized in the AASHTO MEPDG software. Relaxation modulus (E(t)) was evaluated from complex modulus using analytical interconversion procedure (Schapery and Park 1999). Two field aging levels were simulated using GAM, three years (3 Year Aged) and six years (6 Year Aged). The unaged and aged relaxation modulus mastercurves are shown in Figure 5. Notice that in case of aged mastercurves the material behavior is graded through the pavement thickness, showing stiffer and less relaxant behavior near the pavement surface.

Low temperature cracking behavior is evaluated using the VFGM finite element method presented earlier. In addition to the VFGM capabilities, several other computational models have been incorporated in the analysis, such as, cohesive zone fracture model (Song et al. 2006) and temperature dependent thermal expansion and contraction coefficients (Marasteanu et al. 2007, Dave et al. 2008). A critical conditions approach was utilized in the present study, whereby critical low temperature events were identified and simulated; e.g., winter days involving a very high cooling rate and/or a very severe low temperature event. MnROAD cell 03 experienced a critical cooling event during the 1st – 2nd February, 1996 and MnROAD cell 34 had a critical event during the 30^{th} – 31^{st} January 2004. Detailed descriptions of the model and the thermal boundary conditions can be found in Dave et al. (2008).

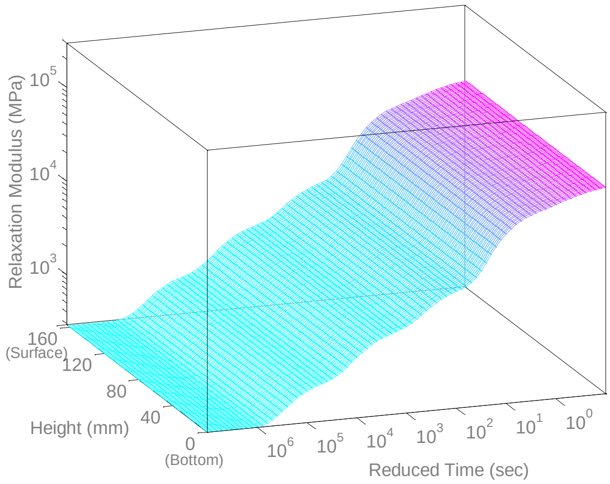
3.2 Simulation Results

The thermal cracking modeling approach utilized in this study simulates the formation and downward propagation of a single crack during the critical cooling event. The cracking potential is evaluated by computing the extent of damage (softening) and cracking (complete material separation) through the thickness of asphalt concrete. Previous studies have demonstrated a good correlation between this crack/damage depth and cracking potential of the pavement (for example, Paulino et al. 2006). For the different pavement aging levels and sections simulated in this study the simulation results are presented in Table 1. For comparison purposes the observed field cracking for the test cells is also included in the table. The observed cracking is based on crack counts performed in 2005, which corresponds to thirteen and six years of service life and associated aging gradients developed over this time frame for the respective sections. The lowest pavement temperature reached during the critical cooling event for each section is also listed in Table 1.

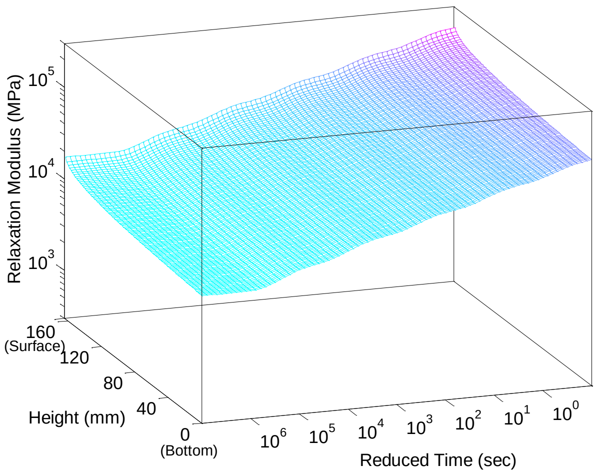
The effect of aging on simulated cracking behavior is evident from the results shown in Table 1. The predicted pavement response under a critical cooling event appears to be consistent with the observed field cracking. For instance, in Cell 03, it is demonstrated that the critical cooling event would produce significant damage and downward cracking due to the effects of thermally induced stress on the aged pavement system. Although the current model has only been applied to a single critical event, it is clear that this section is vulnerable to cracking under thermal cycling in this climate combined with the additional stress imposed by vehicular traffic. As a result, the observed moderate cracking level (36.4m of transverse cracking per 100m of pavement length) is not unexpected, given the propensity for cracking under a critical cooling event as predicted in the current model. Also notice that at the time of crack count Cell 03 had been in service for thirteen years however, due to limited scope of this study the greatest aging level studied was limited to six years. For Cell 34, the pavement section was not predicted to incur any discrete cracking and even very little damage during the critical cooling event. This corresponds well to the observed field performance, which indicates that negligible transverse cracking had been recorded as of the end of the evaluation period in 2005. More work is needed to fully validate the model developed herein, by predicting cracking over multiple cooling cycles and by considering the effects of moving wheel loads on crack movement.



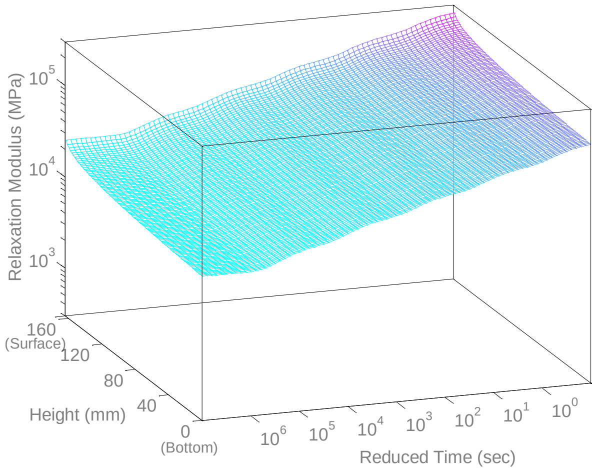
(a) Pavement Sections



(b) Relaxation modulus for unaged section



(c) Relaxation modulus for 3 year aged section



(d) Relaxation modulus for 6 year aged section

Figure 5: Pavement sections and relaxation modulus for unaged and aged conditions at -20ºC (Trends plotted for Section 34 as function of time and depth from pavement surface).

The results of this analysis suggest that field aging gradients should be considered in the prediction of thermal cracking. This is an important finding, as the current MEPDG procedure does not consider aging gradients in thermal cracking predictions. The commonly utilized approach in pavement analysis of assigning homogeneous property to an asphalt layer or considering it by means of a layered approach will lead to under prediction of thermal cracking potential and may require a very high amount of mesh refinement in the vicinity of the pavement surface. Hence, a graded approach such as the VFGM method presented herein provides an efficient and accurate alternative to the conventional layered approach.

Table 1: Simulation results (extent of asphalt layer thickness damaged and cracked), observed field cracking and lowest pavement surface temperature during cooling event.

MnROAD Cell	Aging Level	Simulation Results		Observed Field Cracking (m/100m)	Lowest Pavement Surface Temperature (deg. C)
		Damage (Softening) (%)	Cracking (%)		
03	Unaged	0	0	36.4	-33.8
	3 Year	15	0		
	6 Year	79	58		
34	Unaged	0	0	1.2	-26.2
	3 Year	6	0		
	6 Year	21	0		

4 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A finite element formulation based on a recursive-incremental time integration scheme is presented for viscoelastic functionally graded materials (VFGM). The VFGM finite elements are implemented in commercial software *ABAQUS* using user defined material subroutine. The formulation and implementation are verified through series of examples. Verification examples demonstrate good agreement between present approach and reference solutions. The VFGM analysis procedure has also been applied for low temperature cracking modeling of asphalt pavements. Two pavement sections are studied, each with unaged conditions and two aging levels (3 and 6 years). Simulations results demonstrate that pavement aging has a significant effect on the extent of predicted thermal cracking. A reasonable correlation was noted when comparing the observed field cracking with the predicted damage and cracking for the two sections investigated.

Based on this study it can be concluded that the field aging gradients should be considered when predicting thermal cracking performance of the pavement. This is an important observation as the current MEPDG procedure does not consider aging gradients in thermal cracking predictions. Furthermore, the VFGM finite elements provide an efficient way of considering the material gradations. The commonly utilized approach in pavement analysis of assigning homogeneous property to an asphalt layer or considering it by means of a layered approach will lead to under prediction of thermal cracking potential and may require a very high amount of mesh refinement in the vicinity of the pavement surface.

The present study is limited in terms of the materials and pavement sections studied, and more validation is still needed. More sections and aging levels are recommended to be evaluated in future studies to further explore the effect of aging on thermal cracking as well as other pavement distresses. Traffic effects, multiple temperature cycles, and three dimensional modeling are also needed in future model revisions. Many of these aspects are being considered in an ongoing FHWA Pooled Fund Study on low temperature cracking. Further research is also required in field of non-homogeneous material characterization of asphaltic materials, especially for pavement samples obtained from field sections.

REFERENCES

Branthaver, J.F., Petersen, J.C., Robertson, R.E., Duvall, J.J., Kim, S.S., Harnsberger, P.M., Mill, T., Ensley, E.K., Barbour, F.A., and Schabron, J.F., 1993. *Binder Characterization and Evaluation, Volume 2: Chemistry*. Strategic Highway Research Program, National Research Council, Final Report, SHRP A-368, Washington, DC.

Buttlar, W.G., Paulino, G.H., and Song, S.H., 2006. *Application of Graded Finite Elements for Asphalt Pavements*. Journal of Engineering Mechanics, Vol. 132(3), pp. 240-249.

Chiasson, A.D., Yavuzturk, C., and Ksaibati, K., 2008. *Linearized Approach for Predicting Thermal Stresses in Asphalt Pavements due to Environmental Conditions*. Journal of Materials in Civil Engineering, Vol. 20, pp. 118-127.

Christensen, R.M., 1982. *Theory of Viscoelasticity*. Dover Publications, Inc., Mineola, NY.

Dave, E.V., Braham, A.F., Buttlar, W.G., Paulino, G.H., and Zofka, A., 2008. *Integration of Laboratory Testing, Field Performance Data, and Numerical Simulations for the Study of Low-Temperature Cracking*. Proceedings of the 6th RILEM International Conference on Cracking in Pavements, Taylor and Francis, pp. 369-378.

Dave, E.V., Paulino, G.H., Buttlar, W.G., 2010. *Viscoelastic Functionally Graded Finite Element Method using Correspondence Principle.* Journal of Materials in Civil Engineering (ASCE), *in press.*

Dai, Q. and You, Z., 2009. *Micromechanical Finite Element Framework for Predicting Viscoelastic Properties of Heterogeneous Asphalt Mixtures.* Materials and Structures, Vol. 41(6), pp.1025-1037.

Kim J.H., and Paulino G.H., 2002. *Isoparametric Graded Finite Elements for Nonhomogeneous Isotropic and Orthotropic Materials*. Journal of Applied Mechanics, Vol. 69, pp.502-514.

Marasteanu, M., Zofka, A., Turos, M., Li, X., Velasquez, R., Li, X., Williams, C., Bausano, J., Buttlar, W., Paulino, G., Braham, A., Dave, E., Ojo, J., Bahia, H., Gallistel, A., and McGraw, J., 2007. *Investigation of Low Temperature Cracking in Asphalt Pavements.* Rep. No. 776, MNDOT, Research Services MS 330, St. Paul, MN 55155.

Mirza, M.W., and Witczak, M.W., 1996. *Development of a Global Aging System for Short and Long Term Aging of Asphalt Cements*. Proceedings of the 1995 Conference of the Association of Asphalt Paving Technologists, Maplewood, MN, USA, pp. 393-430.

Mukherjee S, and Paulino G.H., 2003. *The Elastic-Viscoelastic Correspondence Principle for Functionally Graded Materials, Revisited.* Journal of Applied Mechanics, Vol. 70, pp. 359-363.

Muliana A, and Khan K.A., 2008. *A Time-Integration Algorithm for Thermo-Rheologically Complex Polymers*. Computational Materials Science, Vol. 41, pp. 576-589.

Paulino, G.H., Buttlar, W.G., Blankenship, P.B., Wagoner, M.P., Song, S.H., and Dave, E.V., 2006. *Final Report for NSF-GOALI Project - CMS:0219566*. National Science Foundation, Washington, DC, 2006.

Schapery, R.A. and Park, S.W., 1999. *Methods of Interconversion between Linear Viscoelastic Material Functions, Part II - Approximate Analytical Method*. International Journal of Solids and Structures, Vol. 36, pp. 1677-1699.

Song, S.H., Paulino, G.H., and Buttlar, W.G., 2006. *A Bilinear Cohesive Zone Model Tailored for Fracture of Asphalt Concrete Considering Viscoelastic Bulk Material.* Engineering Fracture Mechanics, Vol 73(18), pp. 2829-2848.

Yi S, and Hilton H.H., 1994. *Dynamic Finite Element Analysis of Viscoelastic Composite Plates in the Time Domain.* International Journal for Numerical Methods in Engineering, Vol. 37, pp. 4081-4096.

Zocher, M.A., Groves, S.E., and Allen, D.H., 1997. *A Three-Dimensional Finite Element Formulation for Thermoviscoelastic Orthotropic Media.* International Journal for Numerical Methods in Engineering, Vol. 40(12), pp. 2267-2288.